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Complexity Analysis and Accuracy of Image Recovery Based on Signal Transformation Algorithms

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Abstract: In this paper we compare and analyze the complexity of three functions: Fast Fourier transform (FFT), Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT), used in image transformation. The purpose of all the algorithms is to shift the signal from space or time domain to frequency domain for de-noising or compression. We compare the simulated process time of both one and two dimensional FFT, DCT and DWT (Symlet and Debauches 1) using image and speech signal. The process time is found lowest for FFT and highest for DWT, provided its basis function governs the process time and DCT provide the moderate result. Finally the quality of compressed image under the three mathematical functions are compared, where DWT is found as the best and FFT yields worst result.

Keywords: Basis function, MSE, Butterfly algorithm, process time, confidence level.

I. INTRODUCTION

Enormous number of recent works found pertinent to comparison of three mathematical functions: DFT, DCT and DWT. In [1], DCT, DWT and FFT are applied in lossy image compression technique including theoretical analysis of above three functions. Similar analysis is found in [2] by the same authors, where the image of 256 × 256 for DCT, 100 × 100 for FFT and 720 × 720 for DWT are used and the percentage compression is shown in tabular form.

The final conclusion of both papers is DCT is the best lossy image compression technique. We get the same illusion from JPEG compression algorithm but JPEG 2000 uses DWT. The ECG signal of telecardiology contains large amount of data to be recorded hence its compression is necessary.

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The following mathematical tools of frequency analysis: FFT, DST, DCT, SAPA/FAN and DCT-II are applied for lossless compression of ECG and DCT and SAPA/FAN is found better than other techniques in lossless compression, explained in [3].

In [4], the performance of 32 and 64 point FFT is computed applying multiple RADIX Algorithms. The authors applied the concept of pipeline in FFT, to increase the throughput of system. Simulation of three algorithms: Radix-2, Radix-4 and Radix-8 is used for comparison.

In [5], instead of 'minutiae based approach' the authors applied 'image based approach' in fingerprint identification. Four mathematical tools: Fast Fourier Transform (FFT), Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT) and Gabor Filter has been implemented for feature extraction of fingerprint. The paper also deals with details theoretical analyses of above mathematical tools and the distribution of their coefficients. Similar technique i.e. feature vector of an image of fingerprint is determine using above three mathematical tools in [6], where the percentage of recognition rate is found maximum for DCT and worst for DWT and FFT provides some intermediate value.

Watermarking technique have been applied in several color image by combining DWT (Discrete Wavelet Transform) and DCT (Discrete Cosine Transform). By selecting one color component from RGB (red, green, blue) components of color image, the watermarking technique is done. This methodology is more secured and impactful because the embedded watermarking can only be extracted from the red component or features as discussed in [7]. Combination of DWT and DCT has been used for embedding and extraction copyright protection by using digital watermarking method. This two method DWT+DCT applied on two-dimensional images. This technique works in frequency domain and it is more robust then provides good performance as proposed in [8]. Shivang Ghetia et al. has executed 2-D discrete cosine transform algorithm on GPU. They have used and implemented 2-D DCT with parallel approach on NVIDIA GPU using CUDA (Compute Unified Device Architecture) as discussed in [9].

The Fast Fourier Transform is at every turn used in digital signal processing algorithms. FFT algorithms utilize many applications for example, OFDM, noise reduction, digital audio broadcasting, and digital video broadcasting. It's used to design butterflies for a separate point of FFT discussed in [10].

A similar application of FFT is found in [11], where Cooley and Tukey explained the FFT algorithm relevant to digital signal processing by decreasing the order of complexity of DFT from N2 to Nlog2N. The paper includes the architectures of FFT processor to implement OFDM transmissions, such as a Single path Delay Commutator (SDC), Multipath Delay Commutator (MDC), Single path Delay Feedback (SDF), and Multipath Delay Feedback (MDF). A unique approach to design and perform Fast Fourier Transform(FFT) using Radix-42 algorithm, and how the multidimensional index mapping reduces the complexity of FFT computation have been proposed in [12].

The principal objective of this article is to compare the complexity of three different algorithms of DFT, DCT and DWT both analytically and to verify the time complexity by simulation. We next apply all the algorithms in image compression to observe the quality of images at a glance then then we verify the observation by evaluating mean square error (MSE) between original and recovered image.

The remainder part of the article is composed as section II presents the theory and methodologies of DFT, FFT, DCT and DWT; section III confers the result and discussions based on the theoretical investigation of section II and section IV determines the complete analysis.

II. METHODOLOGY

A. *DFT and FFT*

The Fast Fourier Transform (FFT) is such an algorithm that measures the discrete Fourier transform (DFT) of a sequence, or its counter operation i.e. IDFT is done by IFFT. Fourier transform assign a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.

To produce FFT from DFT, we can separate the an N-point DFT of,

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi nm}{N}} \tag{1}$$

into its even and odd indexed elements like,

$$X(m) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nm} + W_N^m \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2nm} \tag{2}$$

Where, $W_N = e^{-\frac{j2\pi}{N}}$

The sequence $X(m)$ again can be resolve into two parts like,

$$X(m) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{N/2}^{2nm} + W_N^m \sum_{n=0}^{N/2-1} x(2n+1) W_{N/2}^{2nm} = C(m) + W_N^m D(m) \tag{3}$$

and

$$X(m+N/2) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{N/2}^{2nm} - W_N^m \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{\frac{N}{2}}^{2nm} = C(m) - W_N^m D(m) \tag{4}$$

Above process is continued to get 2-point DFT called FFT.

B. *Discrete Wavelet Transform*

In DWT the scaling and shift parameters are $a = 2^m$ and $b = n2^m$ respective, where m and n are integer. The basis function is written as,

$$\Psi_{a,b}(t) = \Psi\left(\frac{t-b}{a}\right) = \Psi\left(\frac{t-n2^m}{2^m}\right) = \Psi(2^{-m}t-n) = \Psi_{m,n}(t) \tag{5}$$

and the DWT of $f(t)$ is,

$$d(m,n) = \frac{1}{2^m}\int_{2^m n}^{2^m(n+1)} f(t)\psi(2^{-m}t-n)dt \tag{6}$$

The DWT of a discrete time sequence $f(n)$ is computed in an alternate way like,

$$W_\varphi(k_0,k) = \frac{1}{\sqrt{M}}\sum_{n=0}^{N-1} f(n), \varphi_{k_0,k}(n) \tag{7}$$

$$W_\Psi(j,k) = \frac{1}{\sqrt{M}}\sum_{n=0}^{N-1} f(n), \Psi_{j,k}(n), \text{ for } j \geq k_0, \tag{8}$$

where $\varphi_{k_0,k}(n)$ and $\Psi_{j,k}(n)$ are the scaling and wavelet basis function, k_0 is the arbitrary staring scale, $W_\varphi(k_0,k)$ and $W_\Psi(j,k)$ are the approximation and detail coefficients.

C. *2D DWT*

One scaling function $\varphi(x, y)$ of 2D DWT and three wavelet functions $\psi^i(x,y);\ i \in \{H,V,D\}$ are used. Let us consider an image $f(p, q)$ of size $M \times N = 2^J \times 2^L$. The DWT is expressed as:

$$W_\varphi(k_0,m,n) = \frac{1}{\sqrt{MN}}\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} f(p,q), \varphi_{k_0,m,n}(p,q) \tag{9}$$

$$W_\psi^i(j,m,n) = \frac{1}{\sqrt{MN}}\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} f(p,q), \psi_{j,m,n}^i(p,q), \tag{10}$$

where $i \in \{H,V,D\}$

Usually $k_0 = 0$ as the convention, $M = N = 2^J$, $j = 0, 1, 2,..,$ J-1and $m = n = 0, 1, 2, …, 2^j$-1.

The scaling and wavelet basis function are expressed as,

$$\varphi_{j,m,n}(x,y) = 2^{\frac{j}{2}}\varphi\left(2^j x - m, 2^j y - n\right)$$

and

$$\Psi_{j,m,n}^i(x,y) = 2^{\frac{j}{2}}\Psi^i\left(2^j x - m, 2^j y - n\right),$$

where their shape depends on type of wavelet like Haar wavelets, Symlets, Daubechies wavelets etc.

D. *Discrete Cosine Transform*

The DCT of a two dimensional signal $f(m, n)$ is expressed as,

$$F(u,v) = \begin{cases} \frac{2}{N}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n) & u = v = 0; \\ \frac{1}{N}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n)\cos\frac{(2m+1)u\pi}{2N}\cos\frac{(2n+1)v\pi}{2N} & 0 \leq u,v \leq N-1; u+v > 0. \end{cases} \tag{11}$$

Here instead of exponential basis function of DFT, DCT uses the cosine function as the basis function.

III. COMPLEXITY ANALYSIS

Fast Fourier transform (FFT) is an algorithm which is used to implement Discrete Fourier Transform (DFT) with lowest complexity, whereas two stage DFT is an intermediate algorithm found better than conventional DFT but worse than FFT. The complexity of conventional DFT is $O(N^2)$, complexity of Butterfly/Radix-2 DFT is $O(N\log_2 N)$ and complexity of 2-stage DFTs is $O(N^2/4)$, the similar complexity is also found for DCT. Two dimensional Haar DWT or 2D Fast Fourier Transform has complexity of $O(4N^2\log_2 N)$. The 2D DWT based on the Mallat wavelet function has the complexity of $O(4MN^2\log_2 N)$, where M represents the vanishing moments number of the mother wavelet used. Actually complexity of DWT depends on basis functions but DFT and DCT has the same basis function hence always shows the same complexity.

IV. ALGORITHM IN COMPUTATION PROCESS TIME AND MSE OF IMAGES

The entire operation of FFT, DCT and DWT on image and signal vector, used in this paper is given below.

1. Read the RGB image $\mathbf{I}_{RGB}$
2. Convert the image $\mathbf{I}_{RGB}$ to grayscale image $\mathbf{I}$ of $M\times N$
3. Convert $\mathbf{I}(x, y)$ to a one dimensional vector $\mathbf{x}(k)$ of $1\times MN$
4. Apply one dimensional DFT, two stage DFT, FFT, DCT and DWT on $\mathbf{x}(k)$
5. Measure CPU time of above operation called experimental process time
6. Compare it with theoretical CPU time derived from big O notation and CPU cycle
7. Apply two dimensional DFT, two stage DFT, FFT,DCT and DWT on $\mathbf{I}(x, y)$
8. Measure CPU time
9. Apply 8×8 block processing operation on image $\mathbf{I}(x, y)$ using a mask of zeros in medium and high frequency region and ones in low frequency region under DCT like JPEG. Compare the original and compressed images.
10. Apply similar operation under FFT and compare the original and compressed images
11. Apply DWT under hard threshold to compress the image $\mathbf{I}(x, y)$
12. Compare the original and compressed image using the formula of mean square error (MSE) as:

$$e=\frac{\sum_{i=1}^{N}\sum_{j=1}^{m}\{x(i,j)-y(i,j)\}^2}{\sum_{i=1}^{N}\sum_{j=1}^{m}x^2(i,j)} \qquad (12)$$

Where, $\mathbf{x}$ represents the original image and $\mathbf{y}$ is the recovered one from the compressed one.

The next part of the paper will deal with the result based on operation of this sub-section.

V. RESULT ANALYSIS

We run the algorithm on the data array of size 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048 and 4096. Every time we determine the process time on the PC (Intel® Core™ i7-8550U CPU, 1.80 GHz, 8.00 GB RAM, CLOCK_PER_SECOND =1000) and plot the results shown in the following fig.1. Incorporating machine parameters with the theoretical complexity (using big O notation) of DFT, we compare the analytical and simulated process time (run time of code) of DFT and found the confidence level of 98% or above. The comparison is made for three cases: in fig. 1(a) butterfly DFT called FFT in fig. 1(b) 2-stage DFT and in fig.1 (c) Conventional DFT.

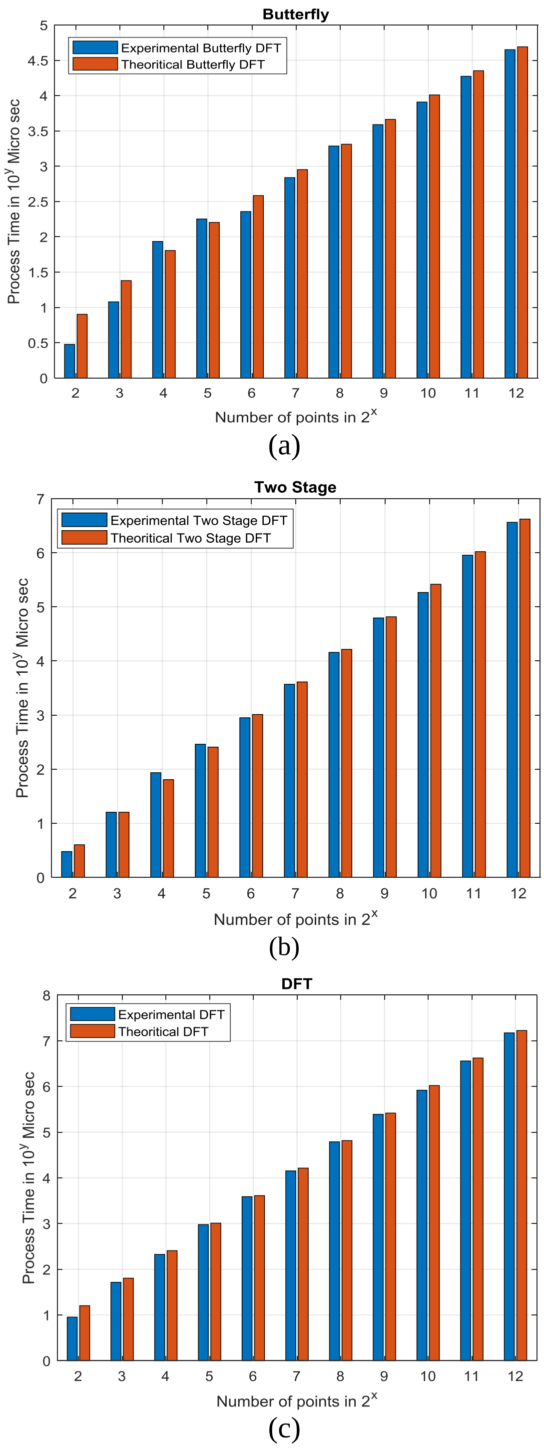


Fig.1 Comparison of theoretical and experimental process time for (a) Butterfly (b) Two stage and (c) Conventional DFT.

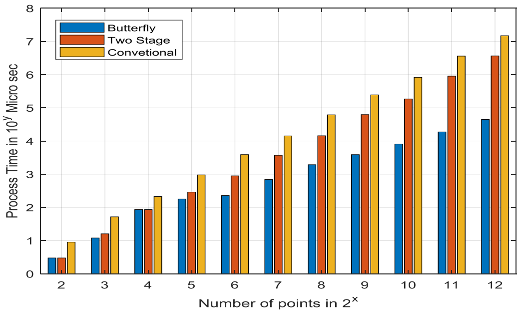


Fig.2 Comparison of process time among three types of DFT operation

After getting high confidence level we only use the simulation results for the rest of the results section. Fig.2 shows the comparison of Butterfly, two stage and conventional DFT, where butterfly seems to be the best, two stage DFT show moderate result and the conventional DFT needs maximum process time. The increment of process time of both two stage DFT and the conventional DFT is linear but Butterfly DFT seems to be non-linear hence the difference in process time for larger input data becomes wider visualized from fig.2.

Next we compare of process time of one dimensional DWT, DCT and FFT as shown in table I. Here we take six images of different size and one speech signal. The image of $N \times M$ is converted to one dimensional vector of $1\times NM$ before applying the algorithms. The process time is found maximum for DWT (also depends on basis function), minimum for FFT and DCT gives intermediate value. Finally two dimensional operation of DWT, DCT and FFT are compared taking the same images as shown in table II, where the relative performance is found same like table I. The process time of two dimensional operation needs more time compared to one dimensional operation for the image of same size. This can be explain from the architecture of a machine in its internal operation on two dimensional signal or matrix.

Table-I: Comparison of process time of one dimensional transform

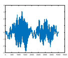
Image	Size	Process Time In seconds DWT (Symlet)	Process Time In seconds DWT (Debauches 1)	Process Time In seconds DCT	Process Time In seconds FFT
	576×787	0.1250	0.1406	0.0781	0.0313
	576×787	0.0938	0.1094	0.0938	0.0313
	512×512	0.1250	0.1563	0.1094	0.0469
	384×512	0.0781	0.1094	0.0625	0.0313
	175×230	0.0781	0.1094	0.0938	0.0313
	4001×1	0.0625	0.0625	0.0469	0.0216
	266×267	0.0938	0.1094	0.1563	0.0938

Table-II: Comparison of process time of two dimensional transform

Image	Size	Process Time In seconds DWT2 (Symlet)	Process Time In seconds DWT2 (Debauches 1)	Process Time In seconds DCT2	Process Time In seconds FFT2
	576×787	0.1719	0.1719	0.1656	0.1094
	576×787	0.2031	0.1719	0.1488	0.1094
	512×512	0.1719	0.1406	0.1575	0.0938
	384×512	0.1719	0.1598	0.1631	0.0938
	175×230	0.0938	0.1094	0.0938	0.0625
	266×267	0.1875	0.2344	0.1563	0.0938

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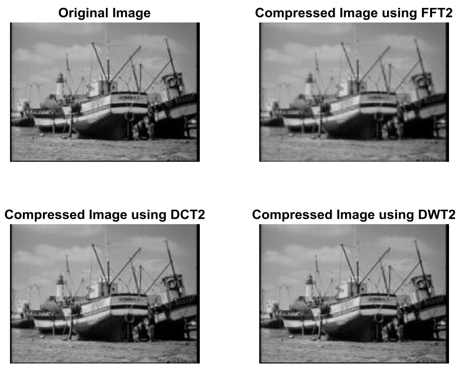
Table III: MSE between original and recovered image

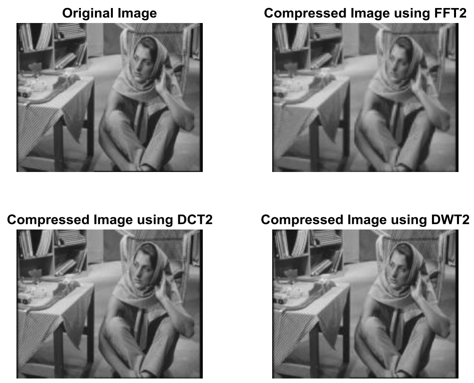
Images	MSE of FFT2	MSE of DCT2	MSE of DWT2	Size of image
Lena	0.0035	0.0023	0.0011	512×512
Barbara	0.0190	0.0122	0.0017	576×720
Boat	0.0092	0.0028	0.0010	576×787
Mandrill	0.0485	0.0206	0.0015	266×267
Onion	0.0096	0.0054	0.0015	135×198
Sangsaptak	0.0217	0.0099	0.0013	175×230

Finally we apply the three mathematical functions in lossy image compression.

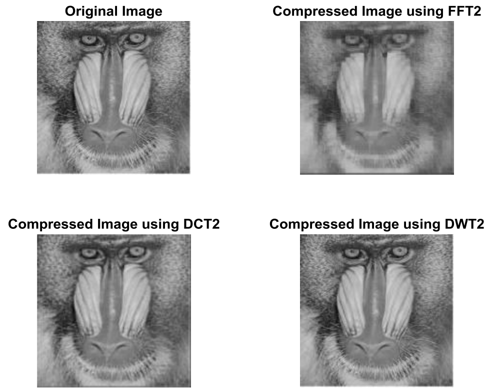
(a)



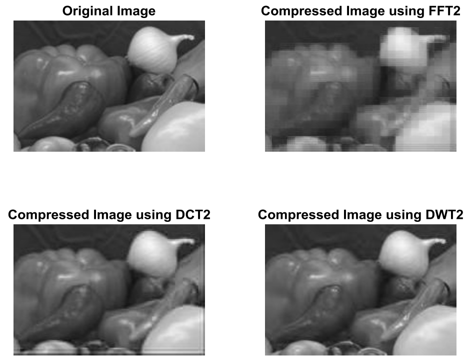
(b)



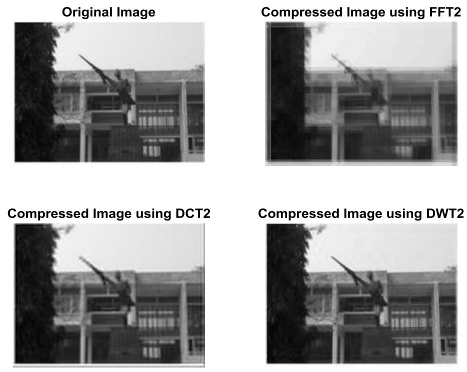
(c)



(d)



(e)



(f)

Fig.3. Comparison of compressed images under three functions (a-f).

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The comparative errors are shown in table III, where DWT2 (Symlet) provides the minimum error and maximum error is found for the case of FFT2. Similar results are also visualized from fig.3, where original image is compared with three compressed images under FFT2, DCT2 and DWT2. The compressed image seems to be the best for DCT2 case at a glance.

VI. CONCLUSION AND FUTURE WORK

The concept of the paper is applicable in development of real time computer vision model for moving object detection. Here the system will sense the presence of an object using spectrum of the moving image to activate the alarm system. In these case, FFT is found best of all in context of process time. Still we have the scope of include adaptive DWT, where among several wavelet basis functions (i.e. Symlet, Debauches, Mallet etc.), we can choose the best one to optimize the accuracy and process time of object recognition/detection.

REFERENCES

1. Teddy Mantoro and Fifit Alfiah, 'Comparison methods of DCT, DWT and FFT techniques approach on lossy image compression,' 2017 International Conference on Computing, Engineering, and Design (ICCED), 23-25 Nov 2017, Kuala Lumpur, Malaysia
2. Fifit Alfiah, Saepudin, Sutarya, Indra Purnama and Ryan Anggara, 'Comparison Methods of DCT, DWT and FFT Techniques Approach on Lossy Image Compression,' International Journal of Computer Techniques, vol. 5, Issue 1, pp.20-24, Feb 2018
3. Hamed Hakkak and Mahdi Azarnoosh, 'Analysis of lossless compression techniques time-frequency-based in ECG signal compression,' Asian Journal of Biomedical and Pharmaceutical Sciences, vol. 9, Issue 66, pp.16-25, Jan 2019
4. Priyanka, R. Bhonde and R. D. Ghongade, 'A Review Paper on Design and Simulation Analysis of 32 and 64 Point FFT Using Multiple Radix Algorithm,' IJCSMC, vol. 5, Issue. 1, pp.198-201, January' 2016
5. Kavita Tewari and Renu L. Kalakoti, 'Fingerprint Recognition and Feature Extraction Using Transform Domain Techniques,' 2014 International Conference on Advances in Communication and Computing Technologies, 0-11 Aug. 2014, Mumbai, India
6. M. P. Dale and M. A. Joshi, 'Fingerprint Matching Using Transform Features,' TENCON 2008, 2008 IEEE Region 10 Conference, 19-21 Nov. 2008, Hyderabad, India
7. Ravinder Singh, Manish Mathuria, Kailash Rathore, Santosh Kumar, 'A Robust Color Image Watermarking using Combination of DWT and DCT,' International Journal of Computer Applications (0975 – 8887), pp.11-14, 4th International IT Summit Confluence 2013.
8. Nawaf Hazim Barnouti, Zaid Saeb Sabri, Khaldoun L. Hameed, 'Digital Watermarking Based on DWT (Discrete Wavelet Transform) and DCT (Discrete Cosine Transform),' International Journal of Engineering & Technology, volume 7(4), pp. 4825-4829, 2018
9. Shivang Ghetia, Nagendra Gajjar, Ruchi Gajjar, 'Implementation of 2-D Discrete Cosine Transform Algorithm on GPU,' International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, vol. 2, pp. 3024-3030, July 2013
10. Mayura Patrikar and Vaishali Tehre , 'Design and Power Measurement of 2 And 8 Point FFT Using Radix-2 Algorithm for FPGA Implementation,' IOSR Journal of VLSI and Signal Processing (IOSR-JVSP), vol. 7, Issue 1, pp. 44-48, Feb. 2017
11. K. Jayaram and C. Arun, 'Survey Report for Radix 2, Radix 4, Radix 8 FFT Algorithms,' International Journal of Innovative Research in Science, Engineering and Technology, vol. 4, Issue 7, pp. 5149-5154, July 2015
12. Aniket Shukla and Mayuresh Deshmukhm, 'Comparative Study of Various FFT Algorithm Implementation on FPGA,' vol.1, Issue 1, pp.19-22, November 2012.

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