Estimation of Recurrent Chaotic Neural Networks Varying Delays through Self-tuning Feedback Control Mechanism

Sathish Kumar Kumaravel, Suresh Rasappan

***Abstract:** In this paper, the estimate the noise of recurrent chaotic neural networks with varying time delays. The design of adaptive synchronization control by means of tracking controller is presented which enables the, exponentially mean square stability of the synchronization error of CRNN system.*

***Keywords:** Recurrent, Chaos, Networks, Delays, Feedback control.*

I. INTRODUCTION

The estimation of chaotic systems plays vital role in many areas such as secure communication, image processing, speech reorganization etc.

Recently the chaotic recurrent neural network (CRNN) with delay system is analyzed by various control techniques such as sample feedback control, back stepping control, nonlinear feedback control etc.

The objective of this paper is to analyze a chaotic neural network with varying time delays involving the noise perturbation. The next section is devoted to this statement of the problem, In section 4, the estimation of CRNN using self-turning control is described section 5,the contains the conclusion

II. PRELIMINARIES AND PROBLEM DESCRIPTION

We use the following notations for a CRNN.

$$dr_c = [-Cr_c + Wf(rc) + W^* f(r_c(t-\beta(t))) + J]\, dt \qquad (1)$$

where the state vector $r_c = (r_{c1}, r_{c2}, r_{c3}, \ldots, r_{cn})^T \in R^n; Cn^{th}$ is n^{th} diagonal matrix consisting of positive real numbers $c_i, i = 1, 2, ..., n,$ and $W = (w_{ij})$ is the weight matrix of order, $W^* = (b_{ij})$ is the delayed weight matrix of order n; $J = [J_1, J_2, \ldots, J_n]^T \in R^n$ is the input vector; $\beta(t)$ the transmission delay; $f(r)$ is activation function. It is reasonable to make the following assumptions for the present study

$H_1: f$, the activation function has the property of roundedness with the fulfillment of the Lipschitz condition $\| f(r_{c1}) - f(r_{c2}) \| \le k_j \; | r_{c1} - r_{c2} |, \; \forall r_{c1}, r_{c2} \in R$

H_2 : $(\beta c)(r_c) \ge 0$ is differential function possessing the property with

$(\beta_c)^* = \max(\beta_c(r_c))$ and $0 \le \beta_c(r_c) \le \sigma < 1, \; \forall \; r_c$.

In Accordance with the master-slave concept for chaotic system can be expressed by means of the equation.

$$\begin{aligned} d\hat{r} &= \left[-C\hat{r}_c + Wf\hat{r}_c + W^* f(\hat{r}(t-\beta_c(t))) + J + u_0(t) \right] dt \\ &\quad + \sum_{i=1}^{n} \left[\sigma_{ij}(t, x_c(t), x_c(t-\beta_c(t))) \right] d\omega_j(t) \end{aligned} \qquad (2)$$

where $\hat{r}_c = (\hat{r}_{c1}, \hat{r}_{c2}, \hat{r}_{c3}, \ldots, \hat{r}_{cn})^T \in R^n$ and $u_0(t)$ is driving signal. This tool having been involved, the expression for the initial condition of the controlled network obtained to be

$$\hat{r}_{ci} = \psi_i \qquad (3)$$

The synchronization error is furnished by the expression

$$x_c = r_c - \hat{r}_c$$

Consequently, the error in the synchronization dynamics between the response system and drive systems provided by (1) and (2) is governed by

$$dx_c = dr_c - \hat{r}_c$$

$$\begin{aligned} dx_c &= \left[-Cx_c + W_c g(x_c) + W_c^* g(x_c(t-\beta_c(t))) - u_0(t) \right] dt \\ &\quad - \sum_{i=1}^{n} \left[\sigma_{ij}(t, x_c, x_c(t-\beta_c(t))) \right] d\omega_j(t) \end{aligned} \qquad (4)$$

Lemma 3.1 : Suppose $\theta_1, \theta_2, \theta_3$ are constant matrices such that $\theta_1 = \theta_1^T$ and $0 < \theta_2 = \theta_2^T$. Then $\theta_1 + \theta_3^T \theta_2^{-2} \theta_3 < 0$ if and only if

$$\begin{bmatrix} \theta_1 & \theta_3^T \\ 0 & -\theta_2 \end{bmatrix} < 0 \qquad (5)$$

Lemma 3.2 : If Σ_1, Σ_2 and Σ_3 are real matrices of appropriate dimensions with $\Sigma_3 > 0$

$$2r^T \Sigma_1^T \Sigma_2 \hat{r}_c \le r_c^T \Sigma_1^T \Sigma_3 \Sigma_1 r_c + \hat{r}_c^T \Sigma_2^T \Sigma_3^{-1} \Sigma_2 \hat{r}_c$$

then for any vectors r_c and $\hat{r}_c$ with suitable dimensions.

Lemma 3.3: Consider a continuous nonlinear system $\dot{r}_c = g(r_c(t), t)$ where $r_c(t)$ is an $n \times n$ vector;

Let us associate the Lyapunov function $V(x,t)$ embedded with the properties

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K. Sathish Kumar, Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R & D Institute of Science and Technology, Avadi, Tamil Nadu, Chennai, 600062, India.

R. Suresh, Department of Mathematics, Vel Tech Rangarajan Dr. Saguntthala R & D Institute of Science and Technology, Avadi, Tamil Nadu, Chennai, 600062, India.

$$\left(\lambda_1 \|r_c\|\right)^2 \le V(r_c,t) \le \left(\lambda_2 \|r_c\|\right)^2, \forall r_c, t \in R^n \times R \tag{6}$$

and

$$\dot{V}(r_c,t) \le -\lambda_3 V(r_c,t) + \lambda_4 e^{-\alpha t}, \forall r_c, t \in R^n \times R \tag{7}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and α are positive scalar constants. The state $r_c(t)$ is observed to be exponentially stable. The state $x_c(t)$ will be exponentially stable with the fulfillment of the properties

$$\|r_c(t)\| \le \left[\left(\frac{\lambda_1}{\lambda_2}\|r_c(0)\|\right)^2 x^{-3\lambda t} + \left(\frac{\lambda_4}{\lambda_1^2} t x^{-3\lambda t}\right)\right]^{\frac{1}{2}} \text{ if } \lambda_3 = \alpha \tag{8}$$

and

$$\|r_c(t)\| \le \left[\left(\frac{\lambda_1}{\lambda_2}\|r_c(0)\|\right)^2 x^{-3\lambda t} + \left(\frac{\lambda_4}{\lambda_1^2(\lambda_3-\alpha)}\left(x^{-\alpha t} - x_c^{-3\lambda t}\right)\right)\right]^{\frac{1}{2}} \text{ if } \lambda_3 \ne \alpha$$

(9)

III. THE ESTIMATION OF CRNN USING SELF-TUNING CONTROL

Controller of the system is expected to posses smart tracking capacity so as to permit the occurrence of tracking capacity convergence. Consequently the condition $t \to \infty$ has to be fulfilled compared to other controller, its find that self turning adaptive control has a special place in the set of controllers with, the adjusting parameters not known for the system, the adaptive controller is working to find the parameter value[11]. Here the self tuning controller should defined by $u_0(t)$

$$u(t) = \gamma P x_c(t) + \frac{1}{2}\rho \frac{\|x_c(t-\beta_c(t))\|^2 \|M_0\|}{R_c\|x_c(t)\|} - \frac{1}{2}\frac{K_c^2 x_c^T(t) R_c^{-1} S_1^T S_1 x t}{K_c\|S_1\|\|x_c^T\|^2 + \varepsilon x_c^{-\alpha t}} \tag{10}$$

Lemma 4.1: Under the conditions of (H_1) and (H_2), if the controller law provided by (10) is followed by the system (7), then the globally synchronization of the controlled slave system (2) will the master system (1), $\lambda_1 = \sqrt{\lambda_{\min}(R_c)}$, $\lambda_2 = \sqrt{\lambda_{\max}(R_c)}$, $\lambda_3 = \frac{\lambda\Omega}{\lambda_{\max}(R_c)}$, $\lambda_4 = \varepsilon$. If $\lambda_3 < 1$, then there exist matrices R_c, S_1, S_2, a diagonal matrix $K_c > 0$ and positive scalars $\gamma > 0$, $\tau > 0$ and $\rho > 0$ satisfying the LMI

$$\begin{bmatrix} -R_c C - C^T R_c + \frac{1+\beta_c^*}{1-\sigma} K_c^T S_2 K_c + K_c^T S_1 K_c + M_c^T \rho M_c & R_c & R_c A & R_c B \\ * & \gamma^{-1} & 0 & 0 \\ * & 0 & -S_1 & 0 \\ * & 0 & 0 & -S_2 \end{bmatrix} < 0$$

(11)

Proof: We deal with Lyapunov functional

$$V(t) = x_c^T(t) R_c x_c(t) + \frac{1}{1-\sigma}\int_{t-\beta_c(t)}^{t} g^T\left(x_c(s)\right) S_2 g\left(x_c(s)\right) ds$$

Using Ito formula, one gets its derivative of $V(t)$ as follows

$$dV(t) = LV(t)dt + 2x_c^T(t) R_c \sigma(x_c(t)), x_c(t-\sigma(t))\, dw(t)$$

where

$$LV(t) = L_t(x_c(t),t) + V_{xc}(x_c(t),t) f(t) + \frac{1}{2}\text{trace}\left[\sigma\left(x_c(t)\right), x_c\left(t-\beta(t)\right) V_{xc} x_c \sigma^T, x_c\left(t-\beta_c(t)\right)\right]$$

$$Vt = \frac{g^T\left(x_c(s)\right) S_2 g(x_c)}{1-\sigma} - g^T\left(x_c\left(t-\beta_c(t)\right)\right) S_2 g\left(x_c\left(t-\beta_c(t)\right)\right),$$

$$V_{xc}\left(x_c(t),t\right) = 2x_c^T(t) R_c$$

and

$$V_{xc}\left(x_c(t),t\right) = 2R_c$$

then the equation (15) becomes

$$\begin{aligned} LV(t) &= \frac{1}{1-\sigma} g^T(x_c(t)) S_2 g(x_c(t)) \\ &- g^T(x_c(t-\beta_c(t))) S_2 g(x_c(t-\beta_c(t))) \\ &+ 2x_c^T(t) R_c\left[-c(x_c) + W_c(g(x_c))\right] + W_c^*\left[g(x_c(t-\beta_c(t))) - u_0(t)\right] \\ &+ \text{trace}\left[\sigma(t, x_c(t), (x_c(t-\beta_c(t))) R_c \sigma^T(t, x_c(t), (x_c(t-\beta_c(t)))\right] \end{aligned}$$

$$\begin{aligned} LV(t) &= 2x_c^T(t) R_c\left[-c(x_c(t)) + W_c(g(x_c(t)))\right] \\ &+ W_c^*\left[g(x_c(t-\beta_c(t))) - u_0(t)\right] \\ &+ \frac{1}{1-\sigma} g^T(x_c(t)) S_2 g(x_c(t)) \\ &- g^T\left(x_c\left(t-\beta_c(t)\right)\right) S_2 g(x_c(t-\beta_c(t))) \\ &+ \text{trace}\left[\sigma(t, x_c(t), (x_c(t-\beta_c(t))) R_c \sigma^T(t, x_c(t), (x_c(t-\beta_c(t)))\right] \end{aligned}$$

(12)

In view of the assumption

$$\begin{aligned} &\text{trace}\left[\sigma(t, x_c(t), (x_c(t-\beta_c(t))) R_c \sigma^T(t, x_c(t), (x_c(t-\beta_c(t)))\right] \\ &\le \rho\left[x_c^T(t) M_c^T M_c x_c(t) + x_c^T\left(t-\beta_c(t) M_c^T\right) M_c x_c\left(t-\beta_c(t)\right)\right] \end{aligned}$$

equation (12) becomes,

$$\begin{aligned} LV(t) &\le e^T\left[-R_c C - C^T R_c\right] x_c(t) + 2x_c^T R_c W_c(g(x_c(t))) \\ &+ 2x_c^T R_c W_c^* g((t-\beta_c(t))) - 2x_c^T(t) R_c u_0(t) \\ &+ \frac{1}{1-\sigma} g^T(x_c(t)) S_2 g(x_c(t)) \\ &- g^T(x_c(t-\beta_c(t)))\, S_2 g(x_c(t-\beta_c(t))) \\ &+ \rho\left[x_c^T(t) M_c^T M_c x_c(t) + x_c^T(t-\beta_c(t)) M_c^T M_c x_c(t-\beta_c(t))\right] \end{aligned}$$

using Lemma (2.3) and taking Σ_3 as the identity matrix

$$\begin{aligned} 2x_c^T R_c W_c(g(x_c(t))) &\le x_c^T R_c W_c S_1^{-1} A^T R_c x_c(t) \\ &+ g^T(x_c(t)) S_1 g(x_c(t)) \end{aligned}$$

we get

$$\begin{aligned} &2x_c^T R_c W_c^*(g(x_c(t-\beta_c(t)))) \\ &\le x_c^T R_c W_c^* S_2^{-1} B^T R_c x_c(t) + g^T(x_c(t-\beta_c)) S_2 g(x_c(t-\beta_c)) \end{aligned}$$

Consequently

$$LV(t) \le x_c^T\left[-R_c C - C^T R_c + R_c W_c S_1^{-1} W_c^T R_c + P W_c^* S_2^{-1} B^T R_c + M_c^T \rho M_c\right] x_c(t)$$
$$+\frac{1}{1-\sigma} g^T(x_c(t)) S_2 g(x_c(t)) - g^T(x_c(t-\beta c)) S_2 g(x_c(t))$$
$$+ x_c^T(t-\beta(t)) M_c^T M_c x_c(t-\beta_c(t)) - 2x_c^T(t) R_c u(t)$$
(13)

Substituting into the equation (13),

$$gT(x_c(t)) S_1 g(x_c(t)) \le x_c^T(t) K_c^T S_1 K_c g(x_c(t))$$
$$gT(x_c(t)) S_2 g(x_c(t)) \le x_c^T(t) K_c^T S_2 K_c g(x_c(t))$$

where K_c is matrix with positive constant entry. The equation (13) gets transformed into

$$\begin{aligned} LV(t) = x_c T[&-R_c C - C^T S + R_c W_c S_1^{-1} W_c^T R_c \\ &+ R_c W_c^* S_2^{-1} B^T R_c + M_c^T \rho M_c + K_c^T S_1 K_c \\ &+ \frac{1}{1-\rho} K_c^T S_2 K_c] x_c(t) \\ &+ x_c^T(t-\beta_c(t)) M_c^T \rho M_c x_c(t-\beta_c(t)) \\ &- 2x_c^T(t)\, R_c u_0(t) \end{aligned}$$

$$\begin{aligned} LV(t) = x_c^T[&-R_c C - C^T R_c + R_c W_c S_1^{-1} W_c^T R_c \\ &+ R_c W_c^* S_2^{-1} W_c^{*T} R_c + M^T \rho M_c + K_c^T S_1 K_c \\ &+ \frac{1}{1-\sigma} K_c^T S_2 K_c + \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c] x_c(t) \\ &+ x_c^T(t-\beta_c(t)) M^T \rho M x_c(t-\beta_c(t)) \\ &- \frac{1}{1-\sigma} K_c^T S_2 K_c + \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c \\ &- 2x_c^T(t) R_c u_0(t) \end{aligned}$$

$$\begin{aligned} LV(t) = -x_c^T[&R_c C + C^T Rc - R_c W_c S_1^{-1} W_c^T R_c \\ &- R_c W_c^* S_2^{-1} W_c^{*T} R_c - M^T \rho M_c - K_c^T S_1 K_c \\ &- \frac{1}{1-\sigma} K_c^T S_2 Kc + \frac{\tau^*}{1-\sigma} K_c^T S_2 K_c] x_c(t) \\ &+ \left\|x_c^T(t-\beta_c(t))\right\|^2 \rho \|M\| \\ &- \frac{1}{1-\sigma} K_c^T S_2 Kc + \frac{\beta_c^*}{1-\sigma} K_c^T S_2 Kc \\ &- 2x_c^T(t) S u_0(t) \end{aligned}$$

now, use the nonlinear adaptive feed back control then,

$$u_0(t) = \gamma R_c x_c(t) + \frac{1}{2}\rho \frac{\|x_c(t-\beta_c(t))\|^2 \|M_c\|}{R_c \|x_c(t)\|} - \frac{1}{2} \frac{K_c^2 x_c^T(t) x(t) R_c^{-1} S_1^T S_1 x_c(t)}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}}$$

then,

$$\begin{aligned} LV(t) \le -x_c^T[&R_c C + C^T R_c - R_c W S_1^{-1} W^T R_c - R_c W^* S_2^{-1} W^{*T} R_c \\ &- M^T \rho M - K_c^T S_1 K_c - \frac{1}{1-\sigma} K_c^T S_2 K_c \\ &- \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c] x_c(t) - \frac{\beta_c^*}{1-\sigma} g^T(x_c(t)) S_2 g(x_c(t)) \\ &+ \frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} \end{aligned}$$

$$LV(t) \le -x^T(t)\theta x_c(t) + \frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} - \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c x_c(t)$$

where

$$\begin{aligned} \theta = R_c C + C^T R_c - R_c W S_1^{-1} W^T R_c - R_c W^* R_c S_2^{-1} W^{*T} R_c \\ - M^T \rho M - K_c T S_1 K_c - \frac{1}{1-\sigma} K_c^T S_2 K_c \\ - \frac{\beta_c^*}{1-\sigma} K_c^T S_2 Kc \end{aligned}$$

$$LV(t) = \frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} - \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c x_c(t)$$

$$dV(t) \le \left[\frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} - \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c x_c(t)\right] dt + 2x_c^T(t) R_c \sigma(x_c(t), x_c(t-\beta_c(t))) d\theta(t)$$

(14)

Expectation, we get considering (14), then

$$\dot{V}(t) = \frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} - \frac{\beta_c^*}{1-\sigma} K_c^T S_2 K_c x_c(t)$$

$$\dot{V}(t) = \frac{K_c \|x_c(t)\|^2 \|S_1\| \varepsilon x_c^{-\alpha t}}{K_c \|S_1\| \|x_c(t)\|^2 + \varepsilon x_c^{-\alpha t}} - \frac{1}{1-\sigma} \int_{t-\beta_c(t)}^{t} g^T(x_c(s)) S_2 g(x_c(s))\, dt,$$

Applying the relation $0 \le \dfrac{ab}{a+b} \le a$ and lemma (2.3), we obtain

$$\dot{V}(t) \le \frac{\lambda_{\min}(\Omega)}{\lambda_{\min}(R_c)}\, v(t) + \varepsilon x_c^{-\alpha t}$$

Invoking Lyapunov stability theorem, it follows that error dynamic system exponentially mean square stable. It is observed that the controlled slave system is globally synchronized with the master system.

IV. CONCLUSION

In this paper a new sufficient condition is proposed to design an exponential mean square stability of the estimation of CRNN with time varying delays. It has been shown how the self-tuning control method can be successfully applied to estimate CRNN. The stability of estimation criteria is expressed in terms of LMIs.

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