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# Selection Criteria of Measurement Matrix for Compressive Sensing Based Medical Image Reconstruction

Ketki C. Pathak, Jignesh Sarvaiya, Anand D Darji, Pooja Panchal

***Abstract*—In this article we design a measurement matrix based on compressive sensing for a medical image in order to achieve a low-cost medical image. In Compressive Sensing based reconstruction of an image, number of samples is smaller than conventional Nyquist theorem suggests. In this paper firstly, we apply DWT(Discrete Wavelet Transform)/DCT(Discrete Cosine Transform) transformations on medical image, and then we use Gaussian random matrices, Bernoulli random matrices, Partial orthogonal random matrices, Partial Hadamard matrices, Toeplitz matrices, and QC_LDPC matrices for medical images, respectively. The compressed medical images are reconstructed with different matching pursuit algorithms: OMP (Orthogonal Matching Pursuit), L1 algorithm and GBP (Greedy Basis Pursuit). Using the same amount of measurement, we select the matrix with the best reconstruction as a measurement matrix for medical images. The reconstruction PSNR values, SSIM values, CR values and reconstruction time were used to compare experimental results. The visual quality of reconstructed medical images is of prime importance for medical images. According to the experiment results, the visual quality of reconstructed medical images with OMP matching pursuit and DWT transform is better than other algorithms so that this paper selects Partial Hadamard matrices with DWT transformation and OMP matching pursuit as medical image measurement matrix.***

***Keywords*—compressive sensing, medical imaging, measurement matrices, recovery algorithm**

## I. INTRODUCTION

Nyquist Sampling Theorem states that "the frequency of sampling signal should be at least twice the signal bandwidth to reconstruct the signal completely". There are four steps in Traditional methods for sampling of signal or images as: sampling of data, compression of sampled data, transmission, and reconstruction as described in Figure 1. But, the sampling frequency in the digital image processing and video processing systems is too high in order that the bandwidth requirement to transmit the signal is also high and it takes too much processing time with conventional signal sampling process. In Compressive sensing (CS), we can reconstruct the image with fewer measurement samples and good visual quality. The signal sampling method based on CS is as shown in Figure 2.

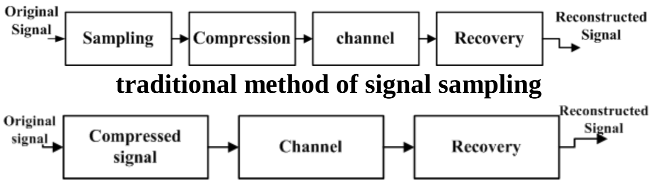


**Fig.2 Sampling method for signal reconstruction**

Theoretical studies show that if we ignore the samples with lower mount of information and select the samples with higher amount of information, then the original signal can be reconstructed with small number of samples [10]. The measurement matrix must satisfy the condition of RIP or incoherence for the reconstruction of the original signal.

In this paper, we use CS for medical image reconstruction to improve the speed and efficiency of compression of an image. The matrix that satisfies the condition of RIP or incoherence is chosen as medical image measurement matrix which may scale back the cost of data acquisition and transmission. The selection of measurement matrix are tested on various medical imaging and performance of Hadamard matrix with Discrete wavelet transform techniques improves the overall performance of compressive sensing based medical image reconstruction. In this proposed analysis we have tested various measurement matrices along with DCT and DWT for sparsity .It is also analyzed with three kind of measurement persuits like OMP, L1 and Gradient based Peruits .The block diagram of proposed work is shown in Figure 3.The overall system of compressive sensing based medical image reconstruction are measured as four different measuring parameters like Peak Signal to Noise Ratio(PSNR) and Encoding time.

In section II we have discussed basic of compressive sensing, section III covers measurement matrices and section IV for result discussion and final section V conclude the result analysis and some future aspect of selection of measurement matrices.

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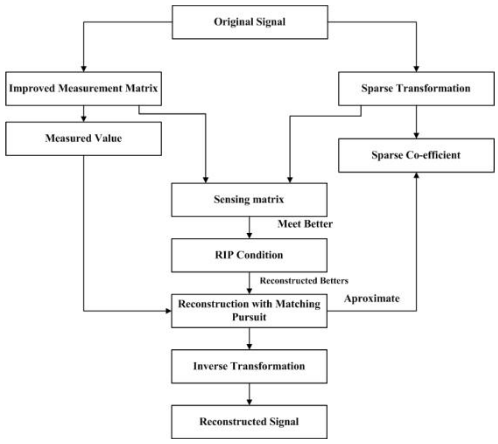


**Figure 1 block diagram of proposed work**

## II. BASIC THEORATICAL ANALYSIS

In compressive sensing sampling and compression is a single step measuring samples in which the maximum value about the signal and removing samples that have minimal value. For this, CS relies on two principles: sparsity and incoherence.

**Sparsity** can be defined as the "information rate" of a signal can be much lower than that suggested by the Nyquist theorem. When the base is selected correctly; many projection coefficients are not possible. The signal only with non-zero coefficients is called as s-Sparse.

Many natural signals are sparse so that they can be represented with small number of samples when expressed in the proper basis Ψ.

Mathematically speaking, we have a vector f € Rn which is expanded in an orthonormal basis $\Psi = [\Psi_1\ \Psi_2\ \Psi_3 \ldots.\ \Psi_n]$ as follows:

$$\emptyset(\tau) = \sum_{t=1}^{v} xi\Psi i(t) \quad (1)$$

Where x is the sequence of coefficient of f, $xi = < f, \Psi_i >$. We can express f as Ψx. When a signal has a sparse expansion, we can ignore the coefficients with small number of information without much loss.

If f is sparse probably the classified magnitudes of the (fi) decay quickly, then f is similar to $f_S$ and, therefore, the error $||x - x_S||l2$ is small. We can ignore many samples with low amount of without much loss.

**Incoherence:** The maximum correlation between any two elements of two different matrices is called as the Coherence. These two matrices can be of two different representation domains. If A is a n x n matrix with $A_1$, $A_2$ ,…., $A_n$ as columns and B is an m x n matrix with $B_1$ $B_2$ .... $B_n$ number of rows. Then coherence μ is defined as:

$$\infty(A,B) = v * \mu\alpha\xi|B\kappa, A\Psi|$$
$$\text{for } 1 \le j \le n$$
$$1 \le k \le m$$

It follows from linear algebra that:

$$1 \le \propto (B, A) \le \sqrt{n}$$

In the CS, low coherence between B and A converts into fewer samples required for reconstruction of signal. Incoherence increases the duality between time and frequency.

The linear measurement process is the basic of compressed sensing, set X(n) as the original signal with N length, X(n) multiplied by the measurement matrix ϕ gets Y(m) with M length (M<N). The measurement process is defined as y=Θs, and Θ = Φψ (M*N), which is called the sensing matrix, the process is shown in figure 4.[8]

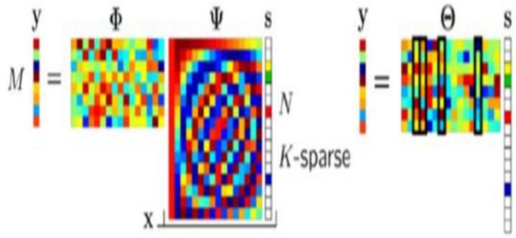


**Figure 2 compressed sensing process[8]**

There are three aspects of compressed sensing theory: sparse representation of signal, construction of measurement matrix and reconstruction of signal.

### *A. Sparse representation of signal*

For sparse representation of image/signal we use a small number of bits by removing the coefficients which are zero or near zero and considering coefficients which carries large amount of information of the image. The necessary condition for the CS theory is sparsity of the signal.

Sparse vectors can be defined mathematically as: Transform coefficient vector of signal a in orthogonal basis of ψ is $\Theta = \psi^T a$, for 0<A<2 and B>0, these coefficient vector satisfies Eq. (1).[1]

$$\|\Theta\|_A \equiv \left(\sum_i |\theta_i|^A\right)^{1/A} \le R \quad (4)$$

The Eq. (4) shows that the coefficient vector is sparse, and θ is the coefficient [2]. The discrete cosine transform, fast Fourier transform, discrete wavelet transform, curvelet transform and Gabor transform are generally used sparse transformation. The discrete wavelet transform and discrete cosine transform are used as sparse transformations for an image in this paper.

### *1. Discrete Wavelet Transform (DWT):*

By using two unidimensional low pass and high pass filters we can change the rows of images. The filtered output is reduced to below two factors. By using the same filters the sub-sampled filtered image is transformed column by column using the same filters, then by a decimation step resulting in four decompositions: LL (Low Pass Low), LH (Low Pass Low), HL Low Pass) and HH (High Pass High Pass). The wavelet transform consists of a cascade of low-pass and high-pass filters, generating an approximation signal.

### *2. Discrete Cosine Transform (DCT):*

The discrete cosine transform (DCT) represents an image as a sum of sinusoids of varying magnitudes and frequencies. In DCT, for a typical image, most of the visually significant information about the image is collected


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in a few coefficients of the DCT; these appear in the upper left corner of the DCT Horizontal frequencies increase from left to right, and vertical frequencies increase from top to bottom. The constant-valued basis function at the upper left is often called the DC basis function, and the corresponding DCT coefficient $B_{00}$ is often called the DC coefficient. This will create problems in image processing.

The One-Dimensional DCT: The definition of DCT for a 1-D sequence of length N is

$$X(V) = \sum_{\xi=\sigma}^{N-1} \emptyset(\xi) x\sigma\epsilon(2\xi + 1)v / 2N$$

for u = 0,1, 2, …., N −1.

Similarly, the inverse DCT is defined as

$$\phi(\xi) = \sum_{\upsilon=\sigma}^{N-1} x(\upsilon)\chi\sigma\epsilon(2\xi + 1)v / 2N$$

for x = 0,1, 2, …, N −1. where α(u) is defined as

$$\alpha(\mu) = \begin{cases} \sqrt{1/\text{N}}, \text{u} = 0 \\ 2/\text{N}, \text{u} \neq 0 \end{cases} \quad (7)$$

Two Dimensional DCT: Two dimensional DCT is given as,

$$C(u) = \alpha(u)\alpha(v) \sum_{y=0}^{N-1} f(x,y) cos\Pi(2x + 1)\, u/ 2N cos\Pi(2y + 1)\, v/2N \quad (8)$$

for u, v = 0,1, 2, …, N −1 and α(u) and α(v) are defined in (7)

Inverse of 2D DCT is given by,

$$f(u) = \alpha(u)\alpha(v) \sum_{y=1}^{N-1} C(u,v) cos\Pi(2x + 1)\, u/ 2N cos\Pi(2y + 1)\, y/2N \quad (9)$$

for x, y = 0,1, 2, …, N −1.

If the lower right values represent higher frequencies and are often small enough to be neglected with little visible distortion then compression is achieved.

*B. Reconstruction algorithm:*

In Reconstruction algorithm the sparse signal p(n) of length N is reconstructed with the measure vector q(m) of length M (M<N). The reconstruction of signal is done by finding the solution of the L0 minimization problem, as shown in Eq. (10).

$$\alpha i = \Phi i^T * r^{n-1} \quad (10)$$

In this paper we use the orthogonal matching pursuit algorithm (OMP) [5], L1 norm method and GBP(Greedy Basis Pursuit) method and algorithms for them are as follow.

*1. OMP (Orthogonal Matching Pursuit):*

OMP is one of the greedy algorithms. Greedy algorithm uses an iterative approach to get an approximate increase of sparse signal in each iteration, by calculating the measured data mismatch. This continues when the coefficient signal reached the desired signal convergence. OMP does the signal recovery quickly with a simple algorithm. The algorithm begins by looking for columns that have the greatest relevance to the measurement. The residual signal will be obtained by subtracting the estimated contribution to the vector signal measurements. Repeat these steps to see the correlation between the columns with the residual signal.

To reconstruct the ideal signal x, we need to determine column ϕ which is measurement matrix used for taking measurement y. In this algorithm we choose columns in greedy fashion. At each iteration, choose column ϕ which is highly correlated with the residual part of y. Then corresponding estimation of x calculated as reducing the resulting residue. After s iteration, algorithm will identify the right values. The algorithm of OMP is as shown below:

- Measurement matrix ϕ of m x n, Y of mx1
- s is sparsity level.
- Initialize the residual $r^0$= Y, the set of indices, $T^0 = \Phi$ and the approximation $X^0$=0.
- Find the most correlated column of Φ with the residual:

$$imax = argmax\,|\alpha i| \quad (11)$$

$$Tn = T^{n-1} * U_{imax} \quad (12)$$

$$X = argmin||Y - \Phi\, T^n X T^n||2 \quad (13)$$

- To obtain the new estimated signal solve the problem of least squares.

$$r^n = Y - \Phi\, X \quad (14)$$

- Calculate the residual

$$r^n = Y - \Phi\, X \quad (15)$$

- The algorithm stops after s iterations.

The flow of OMP method is shown in Figure 5.

*1. L1 minimization:*

One type of recovery is norm minimization technique or basic pursuit. In this technique all the available measurement values considered as set of linear equations. l0- minimization technique has drawback of NP (Non-deterministic polynomial-time hard) hard problem.

So, we go with recovery algorithm l1-norm minimization (Sum of Absolute Difference); the reconstruction x* is defined as x*= Ψ f*, where f* is the solution of the convex optimization program

$$(\|f\|\ell_1 := \sum_i |f_i|) \min_{\tilde{f}\in R^n} \|\tilde{f}\|\ell_1 \text{subject to } y_k = \langle \phi k, \psi \tilde{f} \rangle, \forall\, k \in M \quad (16)$$

That is, among all objects $\tilde{f}= \psi\tilde{x}$ consistent with the data, select only whose coefficient sequence has minimal l1 norm. The linear programming with linear equality constraints are considered in this algorithm [15]. The CS theory uses L1 norm characteristics due to its easy computation, thus offering a far simpler and faster way of estimating sparse signals from very limited number of measurements.

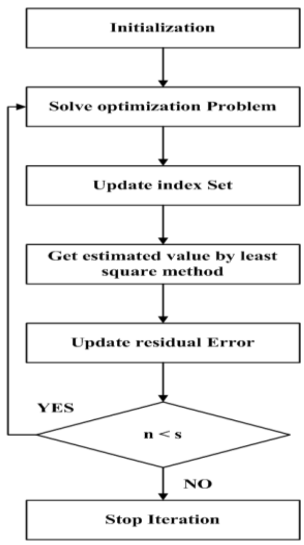



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**Figure 3 Flow graph of OMP algorithm**

*2. Greedy Basis Pursuit*

The GBP algorithm takes x ε $R^d$ as input signal and an over complete dictionary S={$\psi_i$}$_{i=1}^{n}$, where n>d and $\forall_i, \Psi_i \in R^d$ *and* $||\Psi_i||_2 = 1$, and output signal x is represented as a set of indices $I \subseteq \{1,\cdots,n\}$ and a equivalent set of coefficients A={$\alpha_i$}$_{i=1}$ Such that x=$\Sigma_{i=1}\alpha_i\psi_i$.

The GBP algorithm searches the side of conv(D) that intersects x, which is called as $F_x$. In GBP hyperplanes sequences, $H^{(0)}$, $H^{(1)}$, $H^{(2)}$, …,supporting conv(D) is constructed. A set of indices $I^{(t)}$ and a set of coefficients $A^{(t)}$ is maintained at each iteration, defining an approximation to x: $\breve{x}^t = \sum_{i\in I^{(t)}} \propto_i \Psi_i$and a normal vector $n^{(t)}$. The current hyperplane $H^{(t)}$ is defined to have normal $n^{(t)}$ and contain the set $\{\Psi_i\}_{i\in I^{(t)}}$. Each consecutive hyperplane $H^{(t+1)}$ is a rotation of the current hyperplane $H^{(t)}$ determined by $\breve{x}^{(t)}$. The algorithm stops when H(t) contains Fx so that $\breve{x}^{(t)} = x$.[14]

*C. Construction of measurement matrix:*

The measurement matrix should satisfy the RIP or the incoherence. The classification of measurement matrices is as shown in the Fig .6[6]. We use two random unstructured matrices, one random structured matrix, one semi deterministic and one full deterministic matrix.

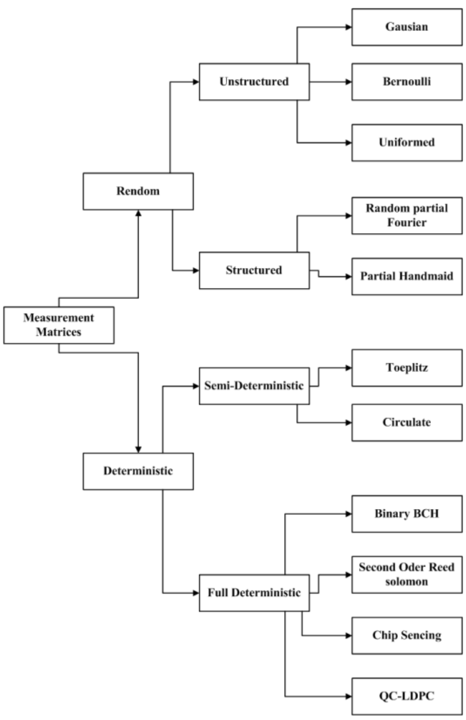


**Figure 4 classification of measurement matrices[6]**

*1. Random Gaussian matrix:*

The elements of a Random Gaussian matrix are independent and normally distributed with expectation 0 and variance $\sigma^2$. The probability density function of a normal distribution is:

$$k \leq C1Nlog\,(L/S) + C2\,log\varepsilon - 1 \qquad (17)$$

Where $\mu$ is the mean of the distribution and $\sigma$ is the standard deviation. The RIP condition is satisfied with probability of minimum $1-\varepsilon$ given that the sparsity satisfy the following formula:

$$k \leq C1Nlog\,(L/S) + C2\,log\varepsilon - 1 \qquad (18)$$

Where S is the sparsity of the signal, N is the number of measurements of the signal, and L is the length of the sparse signal.[6]

*2. Random Bernoulli matrix:*

A random Bernoulli matrix B $\epsilon\mathbb{R}^{M\times N}$ has element values as +1/ $\sqrt{M}$ or $-1$ /$\sqrt{M}$ with equal probabilities. There is two possible outcomes n=0 and n=1 with equal probabilities of p=1/2 and q=1-p=1/2 respecticely. Thus, the probability density function is:

$$f(n) = \begin{cases} 1/2\ ;\ for\ n\ =\ 0 \\ 1/2\ \ ;\ for\ n\ =\ 1 \end{cases} \qquad (19)$$

The RIP condition is satisfoed with the probability which is same as the Random Gaussian matrix.[6]

*3. Partial Orthogonal Random Matrices:*

If square matrix X of order N meets $X^TX=E$, X is named orthogonal matrix. To construct Partial Orthogonal measurement first we generate orthogonal matrix $T$ of dimension N*N, and then we tend to at random choose $M$ rows from $T$, construct the $M*N$ matrix, and unite the $N$ columns. [7]

*4. Random Partial Hadamard matrix:*

The elements of Hadamard measurement matrix are 1 and -1, The columns of which orthogonal. If the transpose of the matrix H($H^T$) of order n is closely related to its inverse then matrix H is said to be a Hadamard matrix.. This can be expressed by:

$$HH^T = NI_N \qquad (20)$$

Where $I_N$ is the identity matrix of size $N \times N$. The RIP condition is satisfied with probability a minimum of $1 - 5\,N - e{-\beta}\,/$ provided $M \geq C_0(1+\beta)K\log N$ where $\beta$ and $C_0$ are positive constants, K is the sparsity of the signal, N is its length and M is the number of measurements [6].

*5. Toeplitz Matrix:*

The Toeplitz matrix $T\epsilon S^{N\times N}$, which is related to a vector $t = (t_{1,2},\ldots t_n)\epsilon S^N$ whose $(i,j) - t$S element is given by:

$$Tij = tj - i \qquad (21)$$

Where $i$, = 1,2,…,$N$. The Toeplitz matrix diagonal are constant i.e. $T_{ij} = T_{i+1\ j+1}$. Thus, the Toeplitz matrix has the following form:

$$T = \begin{bmatrix} t_n & t_{n-1} & \cdots & t_1 \\ t_{n+1} & t_n & \cdots & t_2 \\ \vdots & \vdots & \cdots & \vdots \\ t_{2n-1} & t_{2n-2} & \cdots & t_n \end{bmatrix} \qquad (22)$$

If we have a randomly selected subset $S \subset \{1,\ldots, N\}$ of multiplicity $M \leq N$, the RIP satisfies $\delta_k \leq \delta$ with a high probability provided:

$$N \geq C_\delta K^2\,logN/K \qquad (23)$$

Where $K$ is the sparsity of the signal and $N$ is its length [6].

*6 QC-LDPC Matrix:*

The QC-LDPC with parity check matrix is predicted on cyclic shift matrices, that are made of identity matrix[9]. The parity check matrix of order of m*n of QC-LDPC can be written as

$$H = \begin{bmatrix} I(S_{11}) & I(S_{12}) & \dots & I(S_{1n}) \\ I(S_{21}) & I(S_{22}) & \dots & I(S_{2n}) \\ \vdots & \vdots & \dots & \vdots \\ I(S_{m1}) & I(S_{m1}) & \dots & I(S_{mn}) \end{bmatrix} \quad (24)$$

The algorithm for proposed work is as follow:

i. Perform the transform of the N*N image.
ii. Use the measurement matrix to measure the transformed image.
iii. Find out prediction error for all components.
iv. At receiver side, do prediction on received values.
v. Use the reconstruction algorithm to reconstruct the image.
vi. Apply inverse transform to reconstruct the image.
vii. The image reconstructed will be verified by its PSNR value, SSIM value, CR value and reconstruction time.

## III. RESULT

For experiment we select a medical image of dimension 256 $*$ 256. We use MATLAB 2014a for simulation. The original medical image is measured with random Gaussian measurement matrix, Bernoulli random matrices, Partial Orthogonal random matrices, Partial Hadamard matrices, Toeplitz matrices, and QC-LDPC matrices. The measured medical image is reconstructed by the OMP, L1 and GBP algorithm.

Figure 9-12 show scatter diagram of PSNR and time for 6 different measurement matrices used to measure the medical images. From the scatter diagrams we can see that Hadamard matrix has highest PSNR values compared to others in all the cases excepting the L1_DWT. The PSNR value for the OMP_DCT_hadamard is 34.908 whereas the PSNR value for OMP_DWT_hadamard is infinite.

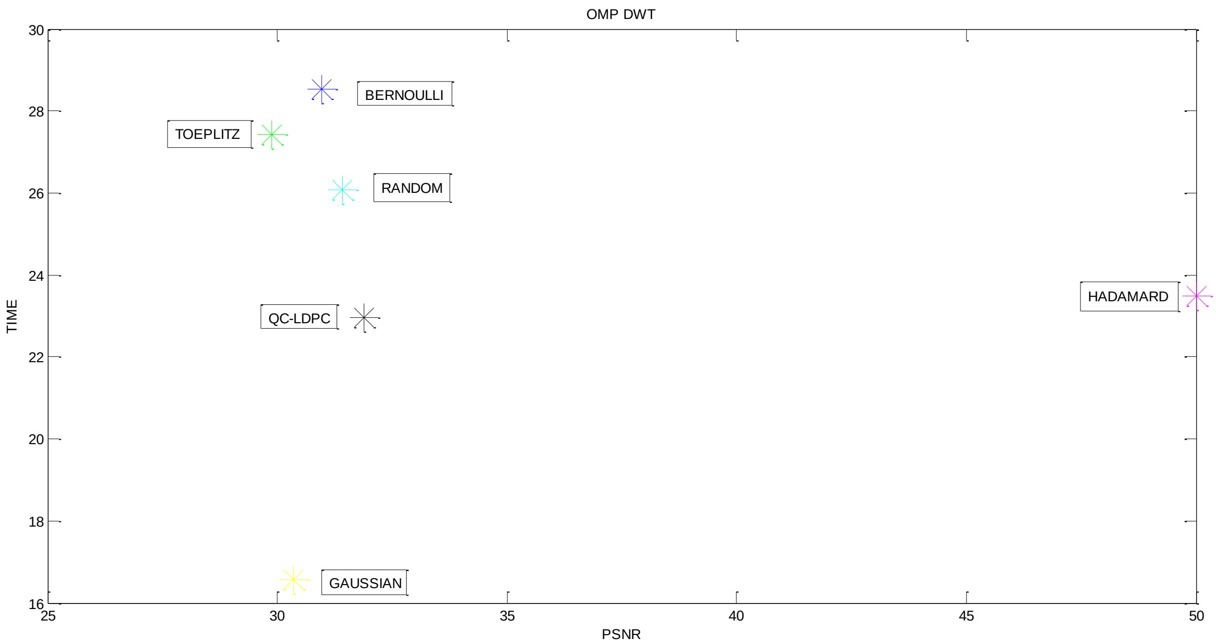


**Figure 5 OMP_DWT**

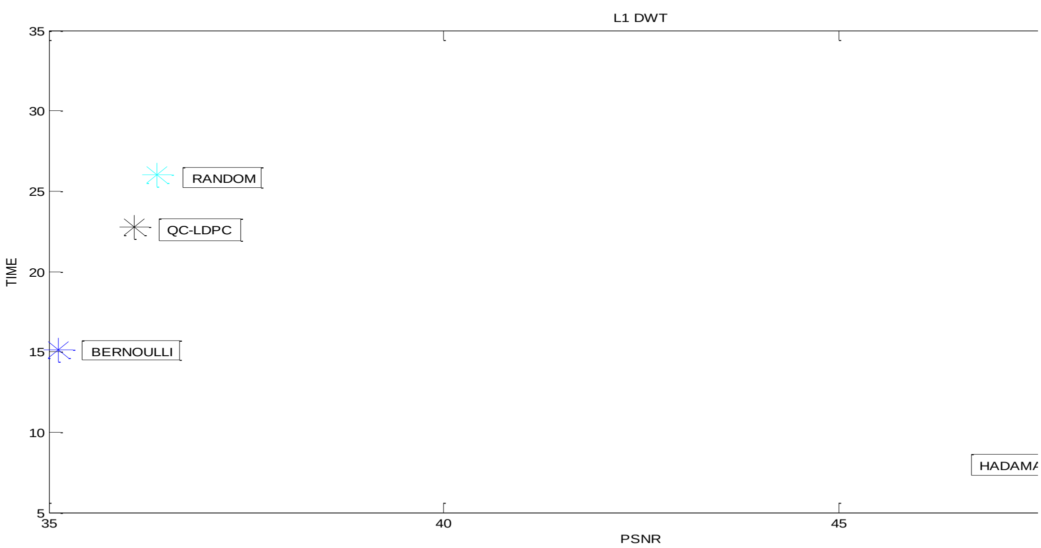


**Figure 6 L1_DWT**


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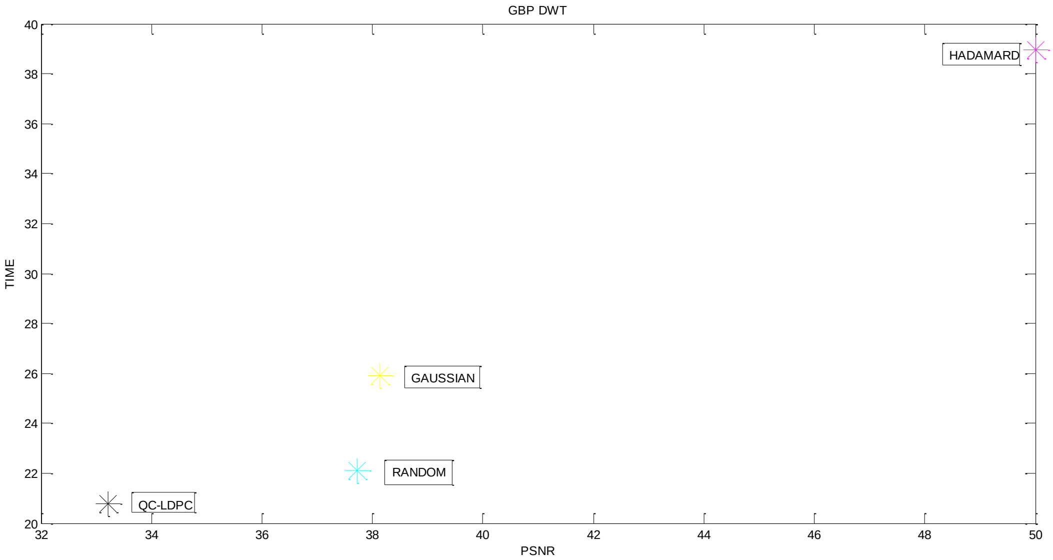


**Figure 7 GBP_DWT**

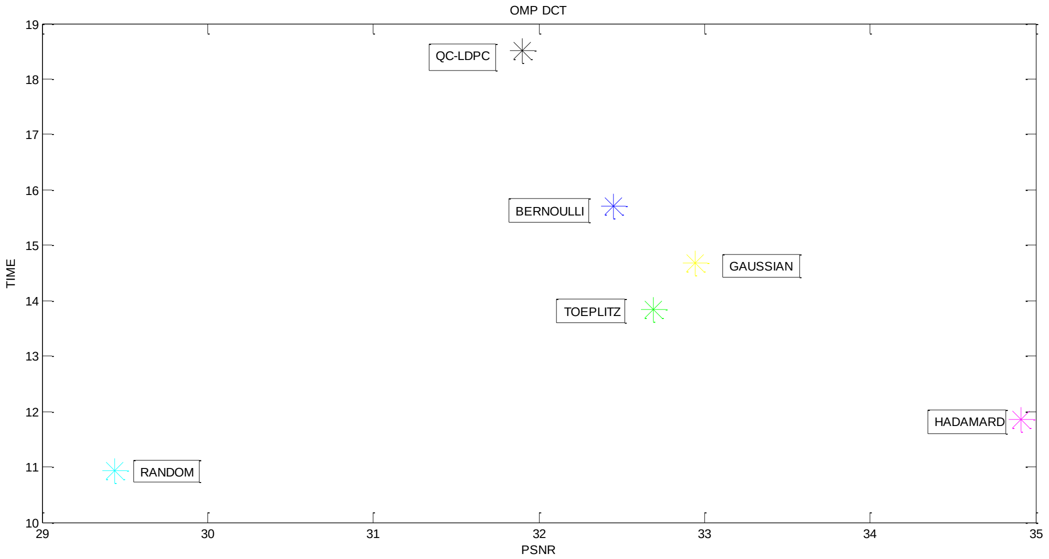


**Figure 8 OMP_DCT**

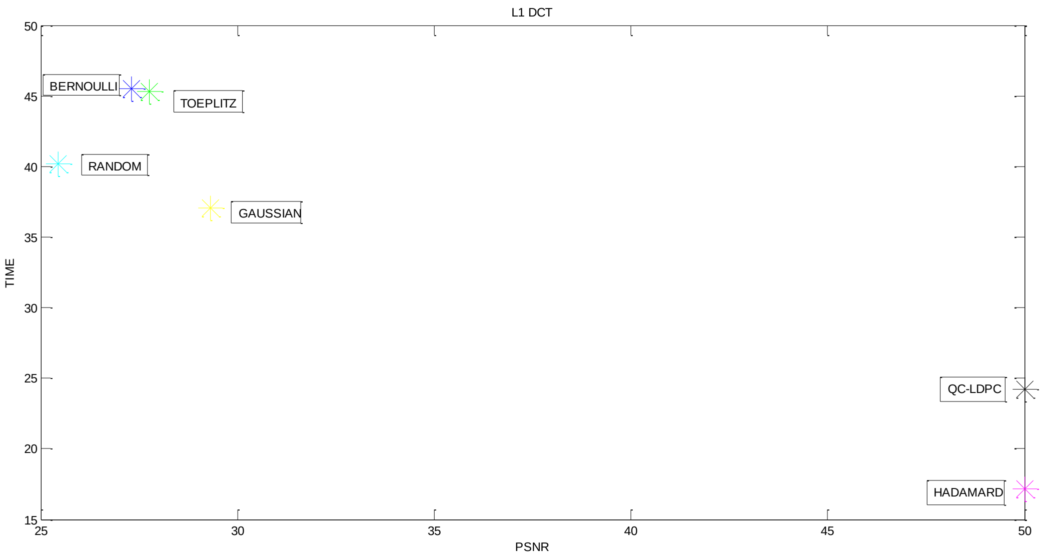


**Figure 9 L1_DCT**

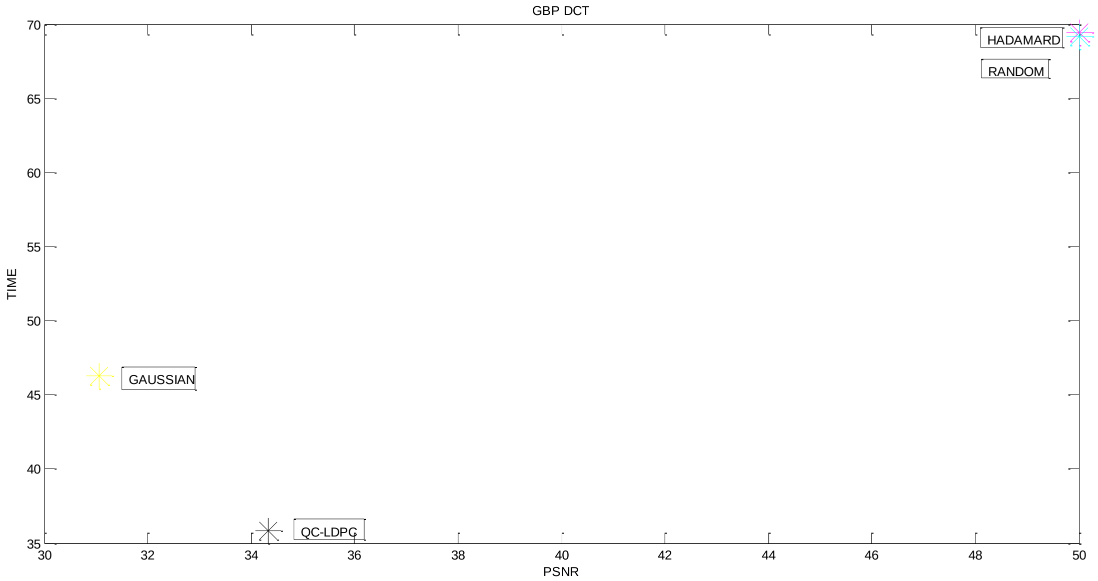


Figure 10 GBP_DCT

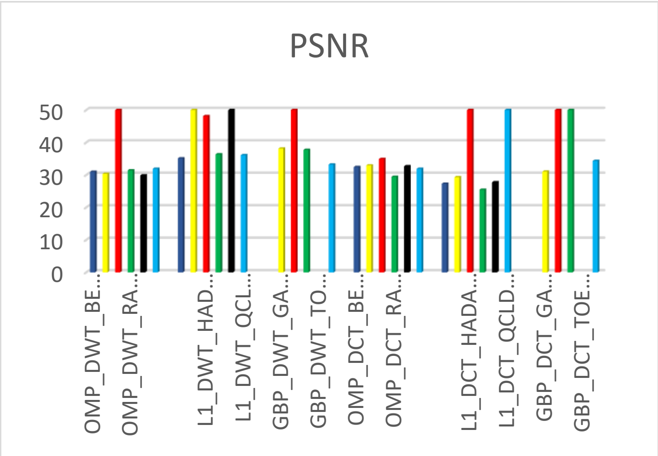


**Figure *15* (a) PSNR comparison**

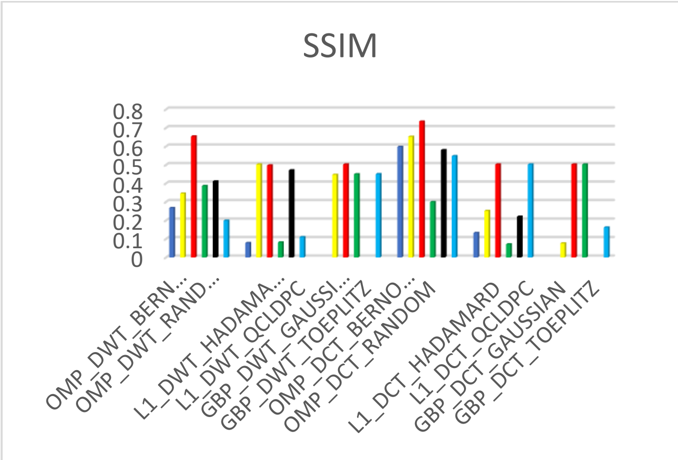


**Figure 15(b) SSIM comparison**

The reconstructed PSNR(peak signal to noise ratio) values, SSIM(structural similarity index measurement) values, CR(compression ratio) values and reconstruction time were used to compare the simulation results. Figures 15 show PSNR, SSIM, CR and reconstruction time bar chart diagrams for 6 different measurement matrices used with different recovery algorithms to reconstruct the medical image.

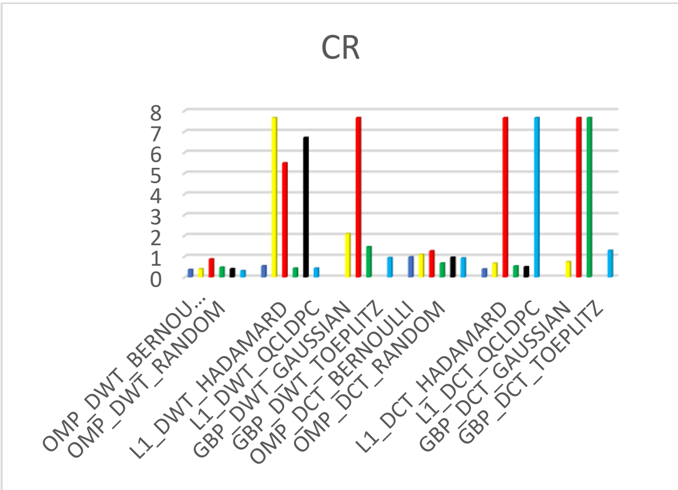


**Figure 15(c) CR comparison**


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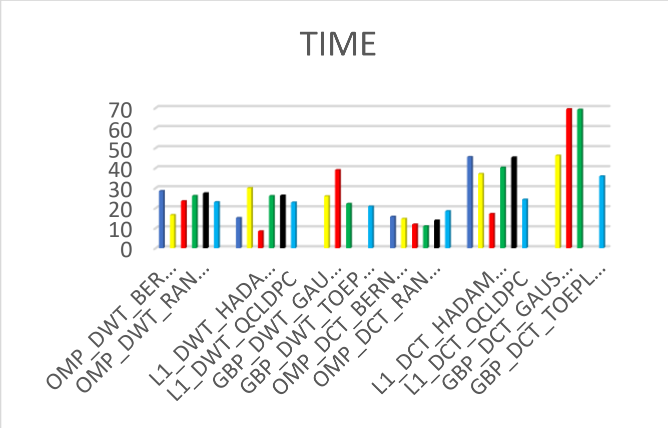


**Figure 15(d) Reconstruction Time comparison**

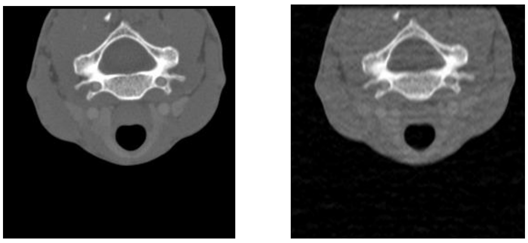

**Figure 16 (a) Original image Figure *16* (b) Output image of OMP_DWT_hadamard**

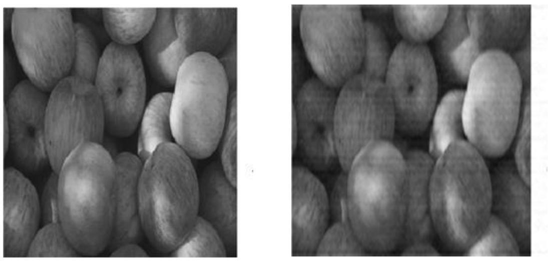

**Figure 11 (a) original image Figure *17* (b) Output image Of OMP_DWT_hadamard**

## IV. CONCLUSION

In this paper firstly, we applies DWT/DCT on medical image as sparse transformation, and then we applies Gaussian random matrices, Bernoulli random matrices, Partial Orthogonal random matrices, Partial Hadamard matrices, QC-LDPC matrices and Toeplitz as measurement matrices. The OMP, L1 minimization and GBP algorithms are used as reconstruction algorithms. We selects Partial Hadamard matrices as medical image measurement matrix with OMP reconstruction algortithm and DWT transformation. Because the visual quality of reconstructed images is of prime importance for medical images and this combination gives the better visual quality compared to others. This measurement matrix is used to compress medical image and to reduce medical image sampling and transport cost.

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