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Parameterization of Unorganized Point Cloud Data for B-Spline Surface Fitting

Vandana Agrawal

***Abstract**: In the present work an algorithm is presented for the parameterization of unorganized point cloud data such that a smooth B-spline surface can be fitted. Points belonging to various surfaces and edges are identified during segmentation. Further edges bounding to segmented region are represented by curves. In the present work initially B-spline curves are constructed with C^1 smoothness by interpolating the measured points lying on the edges. For each segmented region four such curves named as boundary curves are constructed to enclose it. Using these boundary curves Coons surface is constructed which serves as base surface for each segmented region. Each Coons surface is divided into grids and for each measured point the nearest grid vertex is found out. The parameters of this vertex are used as the parameters of the measured point. Finally, an algorithm using an iterative approach is given to further improve the parameterization.*

***Keyword**: parameter, data points, curve, surface*

I. INTRODUCTION

Free-form surface fitting plays an important role in Computer Aided Geometric Design (CAGD). The designers are required to create surfaces from measured points. These measured points are obtained by digitizing the physical part by a measuring device[1,2].

Construction of smooth surface model requires the measured data points to be fitted with a parametric free form surface like Bezier, Coons, B-spline and NURBS[3,4]. Out of all these B-spline stands as one of the most efficient surface representation. It posses many properties [5] such as boundedness, continuity, local shape controllability and invariance to affine transformations that make it very suitable and attractive for surface representation. Due to all these properties it has been widely used for the curve and shape representation[5-12].

Fitting of B-spline surfaces require a neighborhood relationship to be established between the 3D data points as well as a topological meaningful assignments of the parameters. So there have been substantial works on how to derive parameters for fitting these surfaces to 3D scattered non-ordered data set. However, most methods[13,14,15] require user intervention in setting up the patch network by labeling boundary points. Projection of measured points on the base surface formed by approximate characteristic curves of underlying surface was used to find the parameters by Ma & Kruth[16]. These parameters are used directly in solving the equations formed by least square method to compute the control points of the fitted surface. Solution of these equations in the case of surface fitting is very complicated. Eck & Hoppe[17] introduces an automatic construction of a B-spline surface of arbitrary shape but only C^1 continuity was maintained. Cohen[18] proposes a method for parameterization through the geodesics and lines of curvature. But variety of situations were found out where the algorithm was not performing well for example when the surface or part of it was flat, zigzag type of ordering was found out resulting into high curvature fitted surface.

In the present work coordinates of various points on the surface of a telephone receiver were measured by CMM. The obtained data was unorganized point cloud as shown in fig.1.

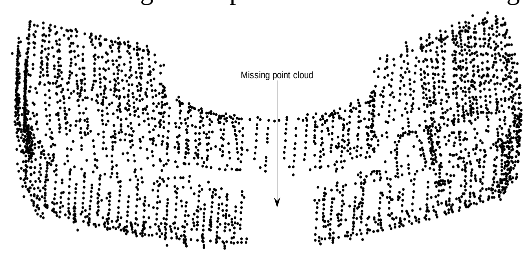

Figure 1 Cloud data obtained after measurement with CMM

Further normal vectors on each of the measured data points were computed by constructing triangular mesh model using Delaunay based region growing algorithm[19].

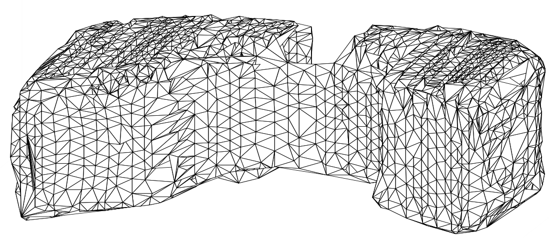

Figure 2 Triangulated model of telephone receiver

Points belonging to various regions and edges were segmented by octree subdivision method[20] by considering the variation in normal vectors at data points. The variations in the unit normal vectors between neighboring points were computed by forming the cells. As the large variations are found out near edges so, edge points were detected and points belonging to different regions were segmented by subdividing the initially formed cells by octree method. The subdivision was done such that small sized cells were on edge points and larger sized cells were on planar areas shown in fig.3.

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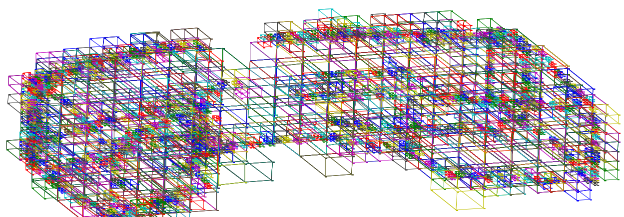

Figure 3 Final grid generated on telephone receiver

Further segmented data points on various regions and edges are shown in figs. 4 and 5 respectively.

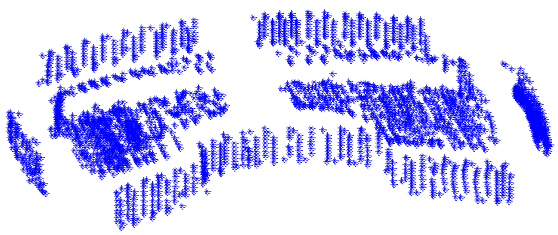

Figure 4 Segmented data of telephone receiver

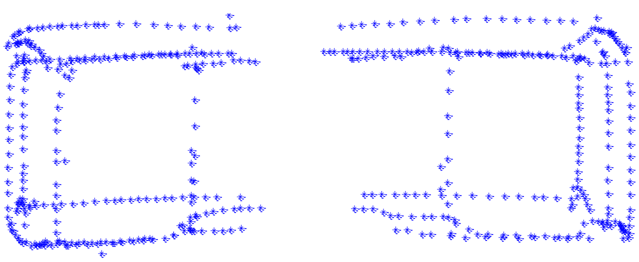

Figure 5 Edge points detected in telephone receiver

In the present work parameterization is done independently for each segmented region. Coons surfaces are used to find the parameters of the measured points to fit the B-spline surface. This paper proceeds as follows. SectionII discusses about B-spline curve and surface fitting. Section III discusses about boundary curve formation. Section IV discusses about formation of Coons surface by blending the boundary curves obtained in sectionIII and parameterization for surface fitting. Section V discusses the results by applying the algorithm on the various segmented regions of telephone receiver. Section 6 gives the conclusion.

II. B-SPLINE CURVE AND SURFACE FITTING

B-spline curves and surfaces are defined by the following equations[3,4] respectively:

$$P(u) = \sum_{i=0}^{n} N_{i,p}(u) P_i \qquad (1)$$

$$P(u,v) = \sum_{i=0}^{n_u} \sum_{j=0}^{n_v} N_{i,p}(u) N_{j,q}(v) P_{ij} \qquad (2)$$

From the above equations it can be seen that a point in the curve given by eq.(1) can be found by the basis functions $N_{i,p}(u)$ for the location parameter u and control points P_i. Similarly a point on the surface in eq.(2) can be found by the basis functions $N_{i,p}(u)$ and $N_{j,q}(v)$ for the location parameters u and v in the two different directions and the control points P_{ij}. These basis functions are computed by Cox-de Boor algorithm [3,4]. Order of the curve in eq.(1) is given by p. In the case of surface order is defined independently for each parametric direction as p and q in the u and v directions respectively in eq.(2). Also knot vectors are required to completely define B-spline curves and B-spline surfaces. These knot vectors are defined for each parametric direction. Therefore in the case of curve it can be defined by $\xi=\{\xi_i \mid i=1,2,\text{-----},n+p+1\}$ and in the case of surface $\xi=\{\xi_i \mid i=1,2,\text{-----},n_u+p+1\}$ and $\zeta=\{\zeta_j \mid j=1,2,\text{-----},n_v+q+1\}$ can be defined as knot vectors for the u and v directions.

To fit a curve by least square method[4] following eq.(3) is used to compute the control points.

$$\left(N^T N\right) P = R \qquad (3)$$

Where

$$N = \begin{pmatrix} N_{1,p}\left(\bar{u}_1\right) & \cdots & N_{n-1,p}\left(\bar{u}_1\right) \\ \vdots & \ddots & \vdots \\ N_{1,p}\left(\bar{u}_{m-1}\right) & \cdots & N_{n-1,p}\left(\bar{u}_{m-1}\right) \end{pmatrix} \qquad (4)$$

If Q_k represent a data point on the curve then $\bar{u}_k$ represents its corresponding parameter. Here, $Q_0, Q_1, Q_2, ---------, Q_m$ are the data points used for constructing the curve and P in eq.(3) is the set of control points used to define the curve. Also, R is given by the following eq.(5)

$$R = \begin{pmatrix} N_{1,p}(\bar{u}_1)R_1 + & \cdots & +N_{1,p}(\bar{u}_{m-1})R_{m-1} \\ & \vdots & \\ N_{n-1,p}(\bar{u}_1)R_1 + & \cdots & +N_{n-1,p}(\bar{u}_{m-1})R_{m-1} \end{pmatrix} \qquad (5)$$

Where R_k is computed by the following eq.(6)

$$R_k = Q_k - N_{0,p}\left(\bar{u}_k\right)Q_0 - N_{n,p}\left(\bar{u}_k\right)Q_m \qquad (6)$$

Here, $k = 1, 2, -----, m-1$.

In the case of surface fitting eq.(2) can be written in the compact form by the following eq.(7)

$$N.P = Q \qquad (7)$$

Here Q represents the collection of coordinates of measured points and P represents the collection of coordinates of control points.

Also, N can be expressed as

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$$N=\begin{bmatrix} N_{u1}(u_1)N_{v1}(v_1) & N_{u1}(u_1)N_{v2}(v_1) & --- & N_{u1}(u_1)N_{vn_v}(v_1) & N_{u2}(u_1)N_{v1}(v_1) & --- & N_{un_u}(u_1)N_{vn_v}(v_1) \\ N_{u1}(u_2)N_{v1}(v_2) & N_{u1}(u_2)N_{v2}(v_2) & --- & N_{u1}(u_2)N_{vn_v}(v_2) & N_{u2}(u_2)N_{v1}(v_2) & --- & N_{un_u}(u_2)N_{vn_v}(v_2) \\ --- & --- & --- & --- & --- & --- & --- \\ N_{u1}(u_m)N_{v1}(v_m) & N_{u1}(u_m)N_{v2}(v_m) & --- & N_{u1}(u_m)N_{vn_v}(v_m) & N_{u2}(u_m)N_{v1}(v_m) & --- & N_{un_u}(u_m)N_{vn_v}(v_m) \end{bmatrix} \quad (8)$$

According to [16] control points in eq.(7) can be found out by minimizing the function F as given in the following eq.(9)

$$\min(F)=(N.P-Q)^T(N.P-Q) \quad (9)$$

Algorithms for solving eq.(9) can be obtained from [21-23]. This gives an idea that if the parameters of the measured points are known then by using eq.(9) control points can be found out which can ultimately be used to define the surface given by eq.(2). So, the problem lies here to find out the parameters u and v for each measured point which is discussed in the present paper.

III. CONSTRUCTION OF BOUNDARY CURVE

Points belonging to different regions were separated and edge points were also extracted as shown in figs. 4 and 5. As edge points must also be at the boundaries of the different segmented regions so the points belonging to boundaries of different segmented regions can be found out with the help of cells formed on these points. This can be done with the help of connectivity check. According to it if any of the vertices of the cells on edge points are common with the vertices of cells formed on a segmented region then those cells on edge points are extracted and the points in these cells are stored as boundary points of the segmented region. Also, these boundary points are stored in sequence and as the boundary curves on a segmented region will make a closed loop so ending point in these stored boundary points must be same as the starting point. These boundary points are divided into four different parts (fig.6) such that each part can be represented by a B-spline curve. This way each segmented region will be closed by four boundary B-spline curves.

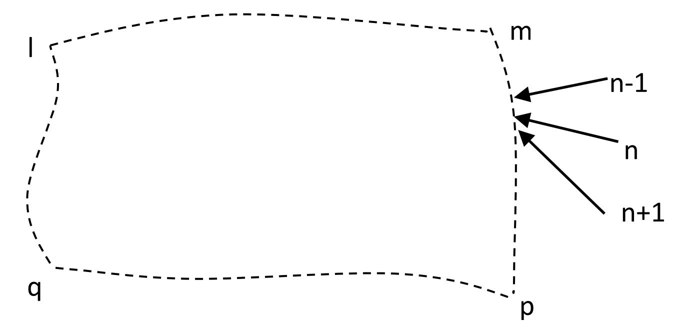


Figure 6 Boundary points enclosing a segmented region

For dividing the boundary points the angle at each point is computed as follows. Let the n[th] point is considered. Let (n-1) and (n+1) are the two adjacent points of point n coming before and after point-n in the sequence(fig. 6). Out of these three points two vectors d_1 and d_2 joining point n to (n-1) and point n to (n+1) can be found out as given by eqs.(10) and (11) respectively

$$d_1=(x_{n-1}-x_n)\hat{\imath}+(y_{n-1}-y_n)\hat{\jmath}+(z_{n-1}-z_n)\hat{k} \quad (10)$$

$$d_2=(x_{n+1}-x_n)\hat{\imath}+(y_{n+1}-y_n)\hat{\jmath}+(z_{n+1}-z_n)\hat{k} \quad (11)$$

Also, angle formed by these two vectors at point n must be

$$\theta_n=\cos^{-1}\left(\frac{d_1.d_2}{|d_1||d_2|}\right) \quad (12)$$

It can be seen from fig.6 that at the points where this angle is larger that portion is smoother and where this angle is smaller the curve shows the turn. So, four points where this angle comes out to be least are selected such that boundary point data for the region are divided at these points. Let four points selected are points m, p, q and l (fig.6) then from point l to point m can be fitted by one boundary curve. Similarly from point m to p, point p to q and point q to l can be fitted by three other boundary curves.

IV. BASE SURFACE CREATION AND PARAMETERIZATION

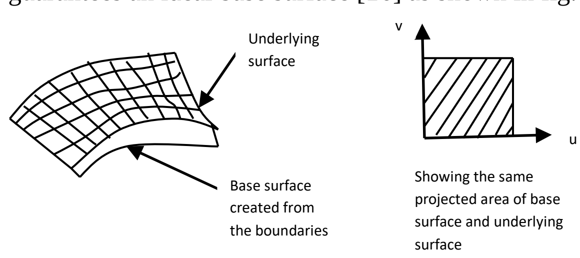
In parametric representation of the surfaces each point on the surface is expressed as a function of two parameters u and v. This shows that by parametric representation each point on the surface can be represented by two variables u and v in place of three variables which are the three coordinates x, y and z. The problem lies over here is to allocate u and v values to various points such that if a function is used to represent the coordinates x, y and z in terms of its parameters then it must truly represent the surface. The basic idea in the present work is to fit a simplified surface on the data points initially which can be called as base surface. Also, this base surface is represented in terms of parameters u and v. After this if the data points are projected on this base surface normally then the parameters of the projected point can be treated as the parameters of the data points. So, the whole problem can be divided into two parts:

A. Base surface creation

B. Parameter allocation

A. **Base surface creation:**

As mentioned above that the base surface is represented in terms of parameters. So, any surface like plane, sphere, cylinder or a free form surface which can be represented in terms of parameters u and v can be treated as the base surface. It should however satisfy the conditions of unique local mapping property and smoothness & closeness of base surface [16]. Also, as parameterization of the base surface directly affects the parameterization of the fitted surface. So, selection of the base surface is very important in view of the parameterization for fitting a surface. It has been found that a base surface defined from four approximate boundaries usually automatically guarantees an ideal base surface [16] as shown in fig.7.



(a) **(b)**

Figure 7 (a) Showing the underlying surface and base surface created from the four boundaries (b) The projected area of underlying surface and base surface in u-v plane

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If the four boundary curves P(u,0), P(u,1), P(0,w) and P(1,w) are known and a bilinear blending function is used for the interior of the surface patch, a linear Coons surface is obtained. In the matrix form it can be shown compactly [3] by eq.(16)

$$Q(u,w) = [1-u \quad u \quad 1]\begin{bmatrix} -P(0,0) & -P(0,1) & P(0,w) \\ -P(1,0) & -P(1,1) & P(1,w) \\ P(u,0) & P(u,1) & 0 \end{bmatrix}\begin{bmatrix} 1-w \\ w \\ 1 \end{bmatrix} \qquad (16)$$

Here P(0,0), P(0,1), P(1,0) and P(1,1) are the end points of the boundary curves shown in fig.8.

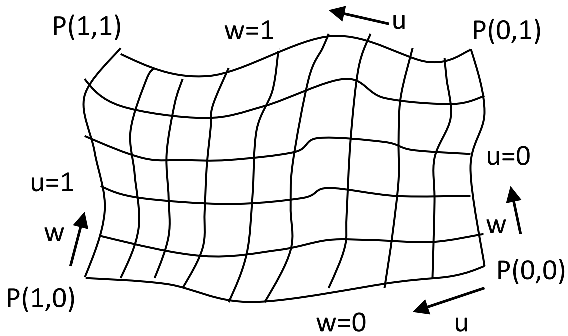

Figure 8 Coons surface showing the four corner points and the boundary curves

B. Parameter allocation

When the base surface is available, the parameterization is realized by projecting the measured points onto the base surface [3]. A standard way of doing the projection is a two step process.

1. For each point find a good initial guess.
2. Perform Newton iteration until convergence is reached

Unfortunately, this is a very expensive and error prone process which can fail quite often especially for points near the boundaries.

In the present work a very simple technique is given. It is based on the assumption that if (u_i,v_i) are the location parameters of the projected point of $\overline{P_i} = [x_i, y_i, z_i]^T$ (projection is taken in the normal direction to the surface) then the distance (d_i) between the surface point $P(u_i,v_i) = [x(u_i,v_i), y(u_i,v_i), z(u_i,v_i)]^T$ and the digitized point P_i must be minimum as shown in fig.9.

In the present work the normalized base surface is divided into finer grids with the width of each grid in both the directions to be 0.001 as shown in fig.10. The coordinates of each vertex of the grid are stored with their parameters ($Q(u,w)$).

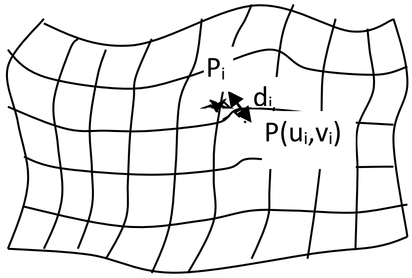

Figure 9 Measured point P_i and projected point $P(u_i,v_i)$ with minimum distance d_i

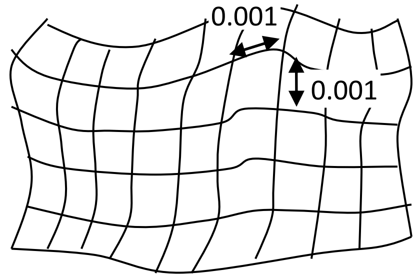

Figure 10 Each grid having the width of 0.001 in both the parametric directions

The distance of a measured point with each vertex of the grid is computed. The vertex at the minimum distance is selected. As described earlier this must be the projection point or very close to projection point of the measured point. The algorithm was applied on the telephone receiver where the maximum length of isoparametric curve was found to be 220mm. This implies that the maximum error possible in this case must be 0.22mm. Further, the results can be improved with the help of Newton's iteration approach as follows.

1. Start with the initial value of projection point ($Q(u^j,w^j)$) for the measured point P_i as found out in the above paragraph.
2. Find out the tangent vectors in u and w directions using eqs.(17) & (18) respectively

$$Q_u(u^j,w^j) = \frac{Q(u^j+\Delta u,w^j)-Q(u^j,w^j)}{\Delta u} \qquad (17)$$

$$Q_w(u^j,w^j) = \frac{Q(u^j,w^j+\Delta w)-Q(u^j,w^j)}{\Delta w} \qquad (18)$$

Here Δu and Δw can be taken as the width of the grid in u & w directions i.e. 0.001.

3. Now, find the direction of normal vector at the base surface at parametric location u^j & w^j by eq.(19)

$$n(u^j,w^j) = \frac{Q_u(u^j,w^j) X Q_w(u^j,w^j)}{|Q_u(u^j,w^j)||Q_w(u^j,w^j)|} \qquad (19)$$

4. Using the normal vector computed by eq.(19) the coordinates of the next iterative value of the projected point can be computed as

$$Q(u^{j+1},w^{j+1}) = P_i + \left|Q(u^j,w^j) - P_i\right|.n(u^j,w^j) \qquad (20)$$

5. Do the convergence check as

If $\left|Q(u^{j+1},w^{j+1}) - Q(u^j,w^j)\right| > \varepsilon$

Then compute the values of u^{j+1} & w^{j+1} by eqs.(21) & (22) as follows

$$u^{j+1} = u^j + \left(Q(u^{j+1},w^{j+1}) - Q(u^j,w^j)\right).\left(Q_u(u^j,w^j) \big/ |Q_u(u^j,w^j)|\right)$$
(21)

$$w^{j+1} = w^j + \left(Q(u^{j+1},w^{j+1}) - Q(u^j,w^j)\right).\left(Q_w(u^j,w^j) \big/ |Q_w(u^j,w^j)|\right)$$
(22)

And repeat the procedure from step 2 otherwise take u^j & w^j to be location parameters for projected point and stop the procedure.

V. RESULTS

Telephone receiver was selected as the object for applying the algorithm. Initially 3299 points on the telephone receiver were measured using coordinate measuring machine. Point cloud is shown in fig.1.

As the measured points are discrete in nature so Delaunay based region growing algorithm [19] was applied to get the triangular mesh model as shown in fig.2. Normal vectors for each measured points were computed using this triangular mesh model. Also, the measured points were divided into cells and subdivision of cells was done using octree method [20] such that small sized cells were formed at edges and large sized cells on the planar region as shown in fig.3. Using this approach the point cloud was segmented into 12 different regions such that each region can be fitted with a single B-spline surface shown in fig.5. Also, the edge points were extracted shown in fig.4. Using the approach described in section 3 initially all the edges belonging to each segmented region were found out. For the same purpose the data points at the edges were arranged in the sequence and divided into four parts as described in section 3 such that each part can be represented by a B-spline curve and each segmented region can be enclosed by four boundary curves. Figure 11 shows all these boundary curves on the telephone receiver with the data points at edges. Also, it can be seen from fig.12 that the curves are very close to measured points on the edges and the measured points on the region for which these boundary curves are computed are coming within these boundary curves. Further, coons surface were fitted using these boundary curves for each segmented region. Two views of the telephone receiver with all these fitted coons surfaces in the form of point grid are shown in fig.13 & 14. In the last parameters for measured points were computed by using the approach given in section 4. Finally using the procedure described from step-1 to step-5 in section 4 the results were further refined by taking the value of ε to be 0.01.

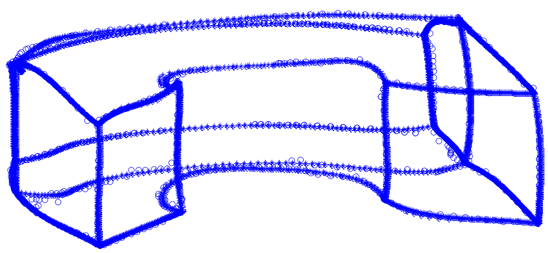

Figure 11 B-spline curves fitted on the edge points of telephone receiver

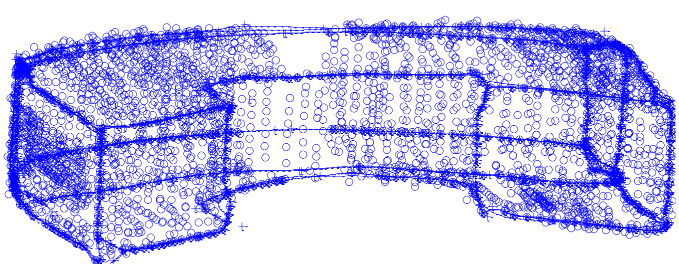

Figure 12 Measured points with curves on edges

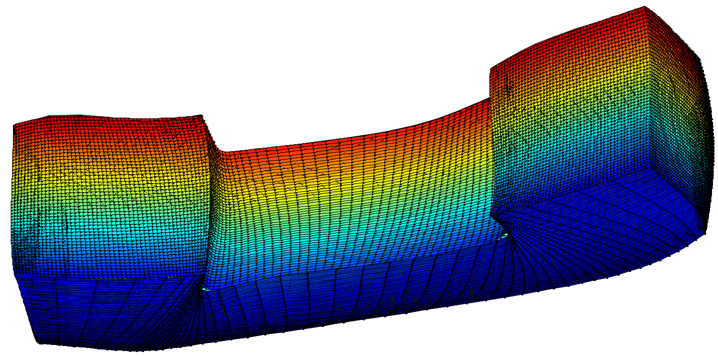

Figure 13 Coons surfaces fitted with the help of curves on edges

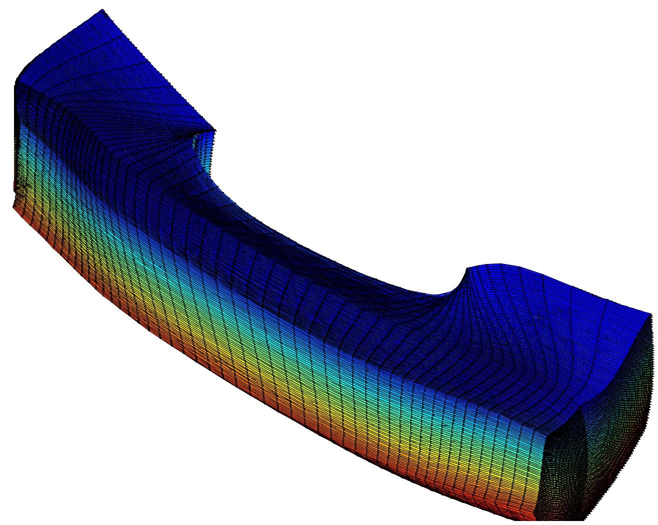

Figure 14 Other view of the telephone receiver with coons surfaces

VI. CONCLUSION

Parameterization of unorganized point cloud data is presented in this work. Initially a segmentation process is required to divide the point cloud into different regions such that each region is separated from the adjacent region by the edge points. Further, the edge points enclosing a segmented region are again divided into four parts such that each part can be represented by a single B-spline curve. In this way four boundary curves enclosing each segmented region is found out. Using these boundary curves a coons surface is deduced as a base surface for each segmented region. Further, the boundary curves were interpolating the most of the points at the edge of the segmented region so the measured points in the segmented region were found to be enclosed by these curves. This eliminates the possibility of having the projected point of the measured point to be outside of the base surface. This fulfills the requirement of an ideal base surface as described in section 4.

Further, the coordinates of the points on the base surface at equal interval (grid points) are found out and stored with their parametric values. For each measured point the closest grid point is selected as the projected point and the parameters of this selected point are stored as the parameters of the measured point. Also, Newton's iterative approach is given in a simplified way to further improve the results. For this again grid points are used to find the derivative values and the stored parameters are used as the initial parameters. If the grid points are taken at very small intervals then the initial value of parameters must be very close to their real values. Also, finding out the derivatives in terms of grid points simplifies the procedure very much and again for small intervals it will give the accurate result.

For the smooth surfaces the algorithm gives very accurate results. However, if segmentation fails to provide simple patches and if intricate details are to be reproduced then one has to go for base surface having more characteristic curves.

REFERENCES

1. Puttre M., "Capturing design data with digitizing systems", Mechanical Engineering 1994,116(1),62-65
2. Wohlers T., "The technology behind 3D digitizing", Computer Graphics World 1997,3(20), 47-54
3. Rogers DF, Adams JA, "Mathematical elements for computer graphics", Tata Mc Graw Hill, Second edition 2002
4. Tiller Wayne, Piegl L, "The NURBS Book", New York, Springer-Verlag 1995
5. De Boor C, "A practical guide to splines", New York, Springer-Verlag 1978
6. Bartels RH, Beaty JC and Barskey BA, "An introduction to splines for use computer graphics and geometric modeling", Morgan Kaufman 1987
7. Piegl L, "On NURBS: A survey", IEEE Trans. Computer Graphics and Applications 1991, 11(5)
8. Yamaguchi F, "Curves and surfaces in computer aided geometric design", New York, Springer Verlag 1988
9. Cohen FS, Wang JY, "Modeling image curves using invariant 3D object curve model- A path to 3D recognition and shape estimation from image contours", IEEE Trans. Pattern analysis and machine intelligence 1994, 16(1),13-21
10. Wang JY, Cohen FS, "3D object recognition and shape estimation from image contours using B-splines, shape invariant matching and neural network", IEEE Trans. Pattern Analysis and machine Intelligence 1994, 16(1), 13-21
11. Cohen FS, Huang Z, Yang Z, "Matching and identification of curves using B-splines curve representation", IEEE Trans. Image Processing 1995, 4(1), 1-10
12. Huang Z, Cohen FS, "Affine invariant moments and B-splines for object recognition from image curves", IEEE Trans. Image Processing 1996, 5(10), 1473-1480
13. Milroy M, Bradley C, Vickers G, Weir D, "G^1 continuity of the B-splines surface patches in reverse engineering", Computer Aided Design 1995, 27, 471-478
14. Krishnamurthy V, Levoy M, "Fitting smooth surfaces to dense polygon meshes" Proceeding SIGGRAPH, Computer Graphics Proc., Ann. Conf. Series 1996, 313-324
15. Andersson E, Andersson R, Boman M, Elmroth T, Dahlberg B, Johansson B, "Automatic construction of surfaces with prescribed shape", Computer Aided Design 1988, 317-324
16. Ma W, Kruth J, "Parameterization of randomly measured points for least squares fitting of B-splines curves and surfaces", Computer Aided Design 1995, 27, 663-675
17. Eck M, Hoppe H, "Automatic reconstruction of B-spline surface of arbitrary topology type", Proceeding SIGGRAPH'96, Comp. Graphics Proc., Ann. Conf. Series 1996, 325-334
18. Cohen FS, "Ordering and parameterizing scattered 3D data for B-spline surface approximation", IEEE Transactions on Pattern analysis and machine intelligence 2000, 22(6)
19. Kuo CC, Yau HT, "A Delaunay based region growing approach to surface reconstruction from unorganized points", Computer Aided Design 2005, 37,825-835
20. Woo H, Kang E, Wang S, Lee KH, "A new segmentation method for point cloud data", Int. J. of Machine Tools and manufacture 2002, 42, 167-178
21. NAG, "Fortran Library Manual Mark" 15 Numerical Algorithm Group Limited 1991, chapter F04
22. De Boor C, "A practical guide to splines" Springer 1978
23. Cox MG, "Linear algebra support modules for approximation and other software" Scientific software systems, Chapman and Hall 1990, 21-29

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