Fekete-Szego¨ Coefficient for the Janowski *A*-*Q*-Spirallike Functions in Open Unit Disk

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***Abstract** In this paper we look at functions which are Janowski α-q-spirallike associated with the m^th^ root transformation using the concept of the q-derivative introduced by Jackson[6] Specifically we look at functions f which are Janowski α-q-spirallike with power senes of the form f(z) = z + $a_2 z^2 + a_3 z^3$ +*

***Keywords** : Convex functions, Janowski α-q-spirallike, Subordination, Hadamard Product, Fekete-Szeg¨o Inequality.*

I. INTRODUCTION

Let E = {z : |z| <1} be the unit disc in the complex plane, and let Ω = {ω : ω analytic in E, ω(0) = 0, |ω(z)| <1, z ∈ E}.

Equivalently (2), may be written as

∞

∂qf(z) = 1 + X [n]q an zn−1, z 6= 0, (3)

Then
$\mathcal{P}(A,B) = \left\{p : p(z) = \frac{1+A\omega(z)}{1+B\omega(z)}, -1 \le B < A \le 1, \omega \in \Omega\right\}$
. Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (1)$$

which are analytic in the open disc E normalized by f(0) = 0, f0(0) = 1. In [6] Jackson introduced and studied the concept of the q-derivative operator ∂q as follows

$$\partial_q f(z) = \frac{f(z) - f(qz)}{z(1-q)}, \ (z \neq 0, 0 < q < 1, \ \partial_q f(0) = f'(0)). \quad (2)$$

n=2

Equivalently (2), may be written as

∞

∂qf(z) = 1 + X [n]q an zn−1, z 6= 0, (3)

where $[n]_q = \frac{1-q^n}{1-q}$, note that as $q \to 1^-$, $[n]_q \to n$.

Ganesan[9] introduced the class $S^*_\alpha(A,B)$ as the class of functions f such that $\frac{\frac{zf'}{f} - i\sin\alpha}{\cos\alpha} \in \mathcal{P}(A,B)$, where α is real and satisfies $|\alpha| < \frac{\pi}{2}$. Now we define the q-analogue of the class as the following:

Definition 1.1. For real α, $\left(|\alpha| < \frac{\pi}{2}\right)$ a function f ∈ A given by (1) is said to be in the class of Janowski α-q-spirallike functions in unit disk if and only if

$$e^{i\alpha} \frac{z\partial_q f(z)}{f(z)} = p(z)\cos\alpha + i\sin\alpha, z \in E, \quad (4)$$

Definition 1.2. If f ∈ A. Then the mth root transform is given by

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$$G(z) = [f(z^m)]^{\frac{1}{m}} = z + \sum_{n=1}^{\infty} c_{mn+1}\ z^{mn+1} . \quad (5)$$

II. Main results

We need the following Lemmas to prove our main results:

Lemma 2.1. [2] If φ ∈ Ω and φ(z) = φ1z + φ2z2 + ..., (z ∈ E) then (6) if μ ≤ −1,

$$|\phi_2 - \mu\phi_1^2| \leq \begin{cases} 1, & \text{if } -1 \leq \mu \leq 1, \\ \mu, & \text{if } \mu \geq 1. \end{cases} \quad (7)$$

For μ <1 or μ >1, the equation holds if and only if, φ(z) = z or one of its rotations. For −1 < μ <1, the equation holds if and only if, φ(z) = z2 or one of its rotations. Equality holds for μ = −1 if and only if $\phi(z) = (z\frac{\lambda+z}{1+\lambda z}), (0 \leq \lambda \leq$ 1) or one of its rotations. While μ = 1, equation holds if and only if $\phi(z) = (-z\frac{\lambda+z}{1+\lambda z}), (0 \leq$ λ ≤ 1) or one of its rotations.

Lemma 2.2. [7] If φ ∈ Ω then $|\phi_2 - \mu\phi_1^2| \leq \max\ \{1, |\mu|\}$, for any complex number μ. The result is sharp for the function φ(z) = z or φ(z) = z2.

Theorem 2.1.

$$\left(\frac{z\partial_q f(z)}{f(z)} - 1\right) \prec \begin{cases} \frac{e^{-i\alpha}(A-B)\cos\alpha z}{1+Bz}, & B \neq 0 \\ e^{-i\alpha} A \cos\alpha z, & B = 0 \end{cases}$$

$f(z) = z + a_2\ z^2 + a_3\ z^3 + ...,\ f(z) \in\ S_\alpha^*(A, B, q)$ if and only if

(8)

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Proof. Let f(z) be an element of $S_\alpha^*\ (A, B, q)$. We define φ(z) by;((A−B)cosαe−iα______f(z) = (1 + Bφ−iα(z)) ______________B , B 6= 0, (9) z eAcosαe , B = 0,

where (1 + $B\phi(z))^{\frac{(A-B)\cos\alpha e^{-i\alpha}}{B}}$ and eAcosα e−iα have the value at z = 0. Then φ(z) is analytic and φ(0) = 0. If we take logarithmic derivative from (9) and after simple calculations,we get (10)

$$\left(\frac{z\partial_q f(z)}{f(z)} - 1\right) = \begin{cases} \frac{(A-B)\cos\alpha e^{-i\alpha} z\partial_q\phi(z)}{1+B\phi(z)}, & B \neq 0 \\ A\cos\alpha e^{-i\alpha} z\partial_q\phi(z), & B = 0 \end{cases}$$

We can easily conclude that this subordination is equivalent to |φ(z)| <1 for all z ∈ E. On the contrary let as assume that there exists z1 ∈ E, such that |φ(z)| attains its maximum value on the circle |z| = r, that is |φ(z1)| = 1. Then when the conditions z1∂qφ(z1) = L(z1), L ≥ 1 are satisfied for such z1 ∈ E (Using Jack's Lemma), we obtain;

,

(11)

which contradicts (10) implying that the assumption is wrong, i.e., |φ(z)| <1 for all z ∈ E. This shows that

(12)

Conversely,

$$f(z) \in S_\alpha^*(A, B, q) \Rightarrow \left(\frac{z\partial_q f(z)}{f(z)} - 1\right) \prec \begin{cases} \frac{e^{-i\alpha}(A-B)\cos\alpha z}{1+Bz}, & B \neq 0 \\ e^{-i\alpha} A\cos\alpha z, & B = 0 \end{cases}$$

(13)

$$\left(\frac{z\partial_q f(z)}{f(z)} - 1\right) \prec \begin{cases} \frac{e^{-i\alpha}(A-B)\cos\alpha z}{1+Bz}, & B \neq 0 \\ e^{-i\alpha} A\cos\alpha z, & B = 0, \end{cases}$$

$$\Rightarrow e^{i\alpha} z\frac{\partial_q f(z)}{f(z)} = \begin{cases} \cos\alpha\frac{1+A\phi(z)}{1+B\phi(z)} + i\sin\alpha, & B\ / \\ \cos\alpha(1 + A\phi(z)) + i\sin\alpha, & B \end{cases}$$

= 0,

(14)

= 0.

This shows that $f(z) \in S^*_\alpha(A, B, q)$.

Now we proceed to establish Coefficient bounds for the mth root transformation,

Theorem 2.2. If and G is the mth root transformation of f given by (1), then

$f = z + \sum_{n=2}^{\infty} a_n z^n, \ f \in S^*_\alpha(A, B, q)$

$$|c_{2m+1} - \mu c^2_{m+1}| \leq \begin{cases} -\left[-\frac{B(A-B)}{([3]_q-1)m)} + [\frac{(A-B)^2}{([3]_q-1)([2]_q-1)m} - \frac{(A-B)^2}{2([2]_q-1)^2 m}] + \frac{(A-B)^2(1-2\mu)}{2m^2([2]_q-1)^2}\right], & \text{if } \mu \geq \sigma_2, B \neq 0 \\ -\frac{B(A-B)}{([3]_q-1)m} + [\frac{(A-B)^2}{([3]_q-1)([2]_q-1)m} - \frac{(A-B)^2}{2([2]_q-1)^2 m}] + \frac{(A-B)^2(1-2\mu)}{2m^2([2]_q-1)^2}, & \text{if } \mu \leq \sigma_1, B \neq 0, \\ \frac{A-B}{([3]_q-1)m}, & \sigma_1 \leq \mu \leq \sigma_2, B \neq 0 \end{cases}$$

where

$$\sigma_1 = \frac{1}{2}\left[1 - \frac{([2]_q - 1)^2 m(2B + ([3]_q - 1))e^{i\alpha}}{([3]_q - 1)(A - B)\cos\alpha} - \frac{2([2]_q - 1)m}{[3]_q - 1} + m\right]$$

and

$$\sigma_2 = \frac{1}{2}\left[1 - \frac{([2]_q - 1)^2 m(2B - ([3]_q - 1))e^{i\alpha}}{([3]_q - 1)(A - B)\cos\alpha} - \frac{2([2]_q - 1)m}{[3]_q - 1} + m\right].$$

Proof. If $f(z) \in S^*(A,B,q)$ then there is an analytic function $\varphi \in \Omega$ of the form (8) such that

$$\frac{z\partial_q f(z)}{f(z)} - 1 = \begin{cases} \frac{e^{-i\alpha}(A-B)\cos\alpha\phi(z)}{1+B\phi(z)}, & \text{if } B \neq 0, \\ e^{-i\alpha}A\cos\alpha\phi(z), & \text{if } B = 0. \end{cases}$$

Further,

$$\frac{e^{-i\alpha}(A - B)\cos\alpha\phi(z)}{1 + B\phi(z)} = \frac{e^{-i\alpha}(A - B)\cos\alpha[\phi_1 z + \phi_2 z^2 + ...]}{1 + B[\phi_1(z) + \phi_2 z^2 + ...]} \quad (15)$$

$$\frac{e^{-i\alpha}(A - B)\cos\alpha\phi(z)}{1 + B\phi(z)} = e^{-i\alpha}(A - B)\cos\alpha[\phi_1 z + (\phi_2 - B\phi_1^2)z^2 + ...].$$

$$\left(\frac{z_1\partial_q f(z_1)}{f(z_1)} - 1\right) = \begin{cases} \frac{(A-B)\cos\alpha e^{-i\alpha}L\phi(z_1)}{1+B\phi(z_1)} = f_1(\phi(z_1)) \in f_1(E), & B \neq 0 \\ A\cos\alpha e^{-i\alpha}L\phi(z_1) = f_2(\phi(z_1)) \notin f_2(E), & B = 0, \end{cases} \quad (16)$$

We have

$$\frac{z\partial_q f(z)}{f(z)} - 1 = e^{-i\alpha}(A - B)\cos\alpha[\phi_1 z + (\phi_2 - B\phi_1^2)z^2 + ...] \quad (17)$$

so that

$$\left(\frac{z + [2]_q a_2 z^2 + [3]_q a_3 z^3 + ...}{z + a_2 z^2 + a_3 z^3 + ...} - 1\right) = e^{-i\alpha}(A-B)\cos\alpha[\phi_1 z + (\phi_2 - B\phi_1^2)z^2 + ...] \quad (18)$$

From (20) and (21),we get

$$a_2 = \frac{e^{-i\alpha}(A - B)\cos\alpha.\phi_1}{[2]_q - 1} \quad (19)$$

$$a_3 = \frac{1}{[3]_q - 1}\left[e^{-i\alpha}(A - B)\cos\alpha(\phi_2 - B\phi_1^2) + \frac{1}{[2]_q - 1}e^{-i2\alpha}(A - B)^2\cos^2\alpha.\phi_1^2\right]. \quad (20)$$

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For function f given by (1) simple computation yields

$$[f(z^m)]^{\frac{1}{m}} = z + \frac{1}{m}a_2 z^{m+1} + \left(\frac{1}{m}a_3 - \frac{1}{2}\frac{m-1}{m^2}a_2^2\right) z^{2m+1} + ... \tag{21}$$

The equations (5) and (24) yields

$$c_{m+1} = \frac{1}{m}a_2 \tag{22}$$

$$c_{2m+1} = \frac{1}{m}a_3 - \frac{1}{2}\frac{m-1}{m^2}a_2^2. \tag{23}$$

by using (22) and (23) in (25) and (26), it follows

$$c_{m+1} = \frac{1}{[2]_q - 1)m} e^{-i\alpha}(A-B)\cos\alpha\phi_1$$

$$c_{2m+1} = \frac{1}{2m}e^{-i\alpha}(A-B)\cos\alpha\left[\frac{2}{[3]_q-1}\left(\phi_2 - B\phi_1^2 + \frac{1}{([2]_q-1)}e^{-i\alpha}(A-B)\cos\alpha\phi_1^2\right)\right] - \frac{1}{2}\left(\frac{m-1}{m^2}\right)\frac{1}{([2]_q-1)^2}e^{-i2\alpha}(A-B)^2\cos^2\alpha\phi_1^2.$$

$$= \frac{1}{2m}e^{-i\alpha}(A-B)\cos\alpha\left\{\frac{2}{[3]_q-1}\phi_2 - \frac{(2B)}{([3]_q-1)}\phi_1^2 + \left(\frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2}\right)e^{-i\alpha}(A-B)\cos\alpha\phi_1^2 + \frac{1}{([2]_q-1)^2 m}e^{-i\alpha}(A-B)\cos\alpha\phi_1^2\right\}.$$

and hence

$$c_{2m+1} - \mu c_{m+1}^2 = \frac{1}{2m}e^{-i\alpha}(A-B)\cos\alpha\left\{\frac{2}{[3]_q-1}\phi_2 - \left[\frac{(2B)}{([3]_q-1)} - N\cos\alpha\right]\phi_1^2\right\},$$

where $N = \frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{([2]_q-1)^2 m}e^{-i\alpha}(A-B)$.

The first result is established by an application of Lemma 2.1. If

$$\frac{2}{[3]_q-1}B - \left(\frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{([2]_q-1)^2 m}\right)e^{-i\alpha}(A-B)\cos\alpha \le -1.$$

then

$$\mu \le -\frac{1}{2}\left[\frac{([2]_q-1)^2 m(2B + ([3]_q-1))e^{i\alpha}}{([3]_q-1)(A-B)\cos\alpha} - \frac{2([2]_q-1)m}{[3]_q-1} + m - 1\right],\ (\mu \le \sigma_1)$$

by Lemma 2.1

$$\left|c_{2m+1} - \mu c_{m+1}^2\right| \le \begin{cases} -\frac{B(A-B)}{([3]_q-1)m} + [\frac{A-B}{([3]_q-1)([2]_q-1)m} - \frac{A-B}{(2[2]_q-1)m}] + \frac{(A-B)^2}{2m^2([2]_q-1)^2}(1-2\mu), & \text{if } B \neq 0 \\ \frac{A^2}{m([2]_q-1)}[\frac{1}{[3]_q-1} - \frac{1}{2}] + \frac{A^2}{2m^2([2]_q-1)^2}(1-2\mu), & \text{if } B = 0. \end{cases},$$

If

$$-1 \le \frac{2}{[3]_q-1}B - \left(\frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{2([2]_q-1)^2 m}\right)e^{-i\alpha}(A-B)\cos\alpha \le 1$$

then

$$\frac{1}{2}\left[1 - \frac{([2]_q-1)^2 m(2B + ([3]_q-1))e^{i\alpha}}{([3]_q-1)(A-B)\cos\alpha} - F\right] \le \mu \le \frac{1}{2}\left[1 - \frac{([2]_q-1)^2 m(2B - ([3]_q-1))e^{i\alpha}}{([3]_q-1)(A-B)\cos\alpha} - F\right]$$

,

where $F = \frac{2([2]_q-1)m}{[3]_q-1} + m$,
and Lemma 2.1 yields

$$|c_{2m+1} - \mu c_{m+1}^2| \leq \begin{cases} \frac{(A-B)}{([3]_q-1)m}, & \text{if } B \neq 0 \\ \frac{A}{([3]_q-1)m}, & \text{if } B = 0. \end{cases},$$

If

$$\frac{2}{[3]_q-1}B - \left[\frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{([2]_q-1)^2 m}\right) e^{-i\alpha}(A-B)\cos\alpha \geq 1$$

then,

$$\mu \geq \frac{1}{2}\left[1 - \frac{([2]_q-1)^2 m(2B-([3]_q-1))e^{i\alpha}}{([3]_q-1)(A-B)\cos\alpha} - \frac{2([2]_q-1)m}{[3]_q-1} + m\right], (\mu \geq \sigma_2)$$

$$|c_{2m+1} - \mu c_{m+1}^2| = \begin{cases} \frac{B(A-B)}{([3]_q-1)(A-B)\cos\alpha} - \frac{(A-B)^2}{([3]_q-1)([2]_q-1)m} + \frac{(A-B)^2}{2m([2]_q-1)} - \frac{(A-B)^2(1-2\mu)}{2m^2([2]_q-1)^2}, & \text{if} B \, / \\ \frac{-A^2}{([2]_q-1)m}\left(\frac{1}{[3]_q-1} - \frac{1}{2}\right) - \frac{A^2(1-2\mu)}{2m^2([2]_q-1)}, & \text{if } B = \end{cases}$$

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t follows from Lemma 2.1 that

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The second result follow by an application of Lemma 2.2

$$|c_{2m+1} - \mu c_{m=1}^2| = \left|\frac{1}{2m}e^{-i\alpha}(A - B\right.$$

$$\leq \begin{cases} \frac{(A-B)}{([3]_q-1)m} \max\left\{1, \left|\frac{(2B)}{([3]_q-1)} - \left(\frac{2}{([3]_q-1)([2]_q-1)} - T\right) e^{-i\alpha}(A-B)\cos\alpha\right|\right\}, & \text{if B 6= 0,} \\ \frac{A}{([3]_q-1)m} \max\left\{1, \left|\left(\frac{2}{([3]_q-1)([2]_q-1)} - T\right)e^{-i\alpha}A\cos\alpha\right|\right\}, & \text{if B = 0.} \end{cases}$$

where $H = \left(\frac{2}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2}\right) e^{-i\alpha}(A-B)\cos\alpha\phi_1^2 + \frac{1-2\mu}{m([2]_q-1)^2}e^{-i\alpha}(A-B)\cos\alpha\phi_1^2$,
and $T = \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{([2]_q-1)^2 m}$.

By putting m = 1, A = 1, B = −1, and α = 0 in the Theorem 2.2, we get the following:

Theorem 2.3. If f ∈ A satisfies $\frac{z\partial_q f(z)}{f(z)} \prec \frac{1+z}{1-z}$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{2}{[3]_q-1} + \frac{4}{([3]_q-1)([2]_q-1)} - \frac{2}{([2]_q-1)^2} + \frac{2}{([2]_q-1)^2} - \frac{4\mu}{([2]_q-1)^2}, & \text{if } \mu \leq \rho, \\ \frac{2}{[3]_q-1}, & \rho \leq \mu \leq \delta \\ -\left[\frac{2}{[3]_q-1} + \frac{4}{([3]_q-1)([2]_q-1)} - \frac{2}{([2]_q-1)^2} + \frac{2}{([2]_q-1)^2} - \frac{4\mu}{([2]_q-1)^2}\right], & \mu \geq \delta \end{cases},$$

where $\rho = \frac{1}{2}\left[1 - \frac{([2]_q-1)^2(-2+([3]_q-1)}{2([3]_q-1)} - \frac{2[2]_q-1)}{[3]_q-1} + 1\right]$, $\delta = \frac{1}{2}\left[1 - \frac{([2]_q-1)^2(-2-([3]_q-1)}{2([3]_q-1)} - \frac{2[2]_q-1)}{[3]_q-1} + 1\right]$.

As q → 1−in the above Theorem we get the following result proved by Annamalai[3]

Corollary 2.1. If f ∈ A satisfies $\frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z}$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu & \text{if } \mu \leq \frac{1}{2}, \\ 1 & \text{if } \frac{1}{2} \leq \mu \leq 1, \end{cases} \tag{24}$$

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□ $-(3-4\mu)$ if $\mu \geq 1$.

By putting m = 1, A = 1, B = 0, and α = 0 in the Theorwe 2.2, we get the following:

Theorem 2.4. If f ∈ A satisfies $\frac{z\partial_q f(z)}{f(z)} \prec 1+z$. Then

$$|a_3-\mu a_2^2| \leq \begin{cases} \frac{1}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{2([2]_q-1)^2}, & if\ \mu \leq \rho, \\ \frac{1}{[3]_q-1}, & \rho \leq \mu \leq \delta, \\ -\left[\frac{1}{([3]_q-1)([2]_q-1)} - \frac{1}{([2]_q-1)^2} + \frac{1-2\mu}{2([2]_q-1)^2}\right], & \mu \geq \delta, \end{cases}$$

where $\rho = \frac{1}{2}\left[2 - \frac{([2]_q-1)^2(-2+([3]_q-1))}{2([3]_q-1)}\right]$, $\delta = \frac{1}{2}\left[2 - \frac{([2]_q-1)^2(-2+([3]_q-1)}{2([3]_q-1)}\right]$.

As q → 1−in the above Theorem we get the following result proved by Annamalai[3]

Corollary 2.2. If f ∈ A satisfies $\frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z}$. Then

$$|a_3-\mu a_2^2| \leq \begin{cases} \frac{1-2\mu}{2} & \text{if } \mu \leq 0, \\ \frac{1}{2} & \text{if } 0 \leq \mu \leq 1, \\ -\left(\frac{1-2\mu}{2}\right) & \text{if } \mu \geq 1. \end{cases} \quad (25)$$

By putting m = 1, A = β, B = 0, and α = 0, 0 ≤ β <1 in the Theorem 2.2, we get the following:

Theorem 2.5. If f ∈ A satisfies $\frac{z\partial_q f(z)}{f(z)} \prec 1+\beta z$. Then

$$|a_3-\mu a_2^2| \leq \begin{cases} \frac{\beta^2}{([3]_q-1)([2]_q-1)} - \frac{\beta^2}{([2]_q-1)^2} + \frac{\beta^2(1-2\mu)}{2([2]_q-1)^2}, & if\ \mu \leq \rho, \\ \frac{1}{[3]_q-1}, & \rho \leq \mu \leq \delta, \\ -\left[\frac{\beta^2}{([3]_q-1)([2]_q-1)} - \frac{\beta^2}{([2]_q-1)^2} + \frac{\beta^2(1-2\mu)}{2([2]_q-1)^2}\right], & \mu \geq \delta, \end{cases}$$

where $\rho = \frac{1}{2}\left[2 - \frac{([2]_q-1)^2([3]_q-1)}{([3]_q-1)\beta} - \frac{2([2]_q-1)}{[3]_q-1}\right]$, $\delta = \frac{1}{2}\left[2 + \frac{([2]_q-1)^2([3]_q-1)}{([3]_q-1)\beta} - \frac{2([2]_q-1)}{[3]_q-1}\right]$.

As q → 1−in the above Theorem we get the following result proved by Annamalai[3] Corollary 2.3. If f ∈ A satisfies $\frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z}$. Then

$$|a_3-\mu a_2^2| \leq \begin{cases} \frac{\beta^2(1-2\mu)}{2} & \text{if } \mu \leq \frac{\beta-1}{2\beta}, \\ 1-\beta & \text{if } \frac{\beta-1}{2\beta} \leq \mu \leq \frac{\beta+1}{2\beta}, \\ -\left(\frac{\beta^2(1-2\mu)}{2}\right) & \text{if } \mu \geq \frac{\beta+1}{2\beta}. \end{cases} \quad (26)$$

By putting m = 1, A = β, B = −β, and α = 0, 0 ≤ β <1 in the Theorem 2.2, we get the following:

Theorem 2.6. If f ∈ A satisfies $\frac{z\partial_q f(z)}{f(z)} \prec \frac{1+\beta z}{1-\beta z}$. Then

$$|a_3-\mu a_2^2| \leq \begin{cases} \frac{2\beta^2}{([3]_q-1)} + \frac{(2\beta)^2}{([2]_q-1)([3]_q-1)} - \frac{(2\beta)^2}{([2]_q-1)([3]_q-1)} + \frac{4\beta^2(1-2\mu)}{2([2]_q-1)^2}, & \text{if } \mu \leq \rho, \\ \frac{\beta+\beta}{[3]_q-1}, & \rho \leq \mu \leq \delta, \\ -\left[\frac{2\beta^2}{([3]_q-1)} + \frac{(2\beta)^2}{([2]_q-1)([3]_q-1)} - \frac{(2\beta)^2}{([2]_q-1)([3]_q-1)} + \frac{4\beta^2(1-2\mu)}{2([2]_q-1)^2}\right], & \mu \geq \delta, \end{cases}$$

where $\rho = \frac{1}{2}\left[2 - \frac{([2]_q-1)^2(-2\beta+([3]_q-1)}{2([3]_q-1)\beta} - \frac{2([2]_q-1)}{[3]_q-1}\right]$, $\delta = \frac{1}{2}\left[2 + \frac{([2]_q-1)^2(2\beta+([3]_q-1)}{2([3]_q-1)\beta} - \frac{2([2]_q-1)}{[3]_q-1}\right]$.

If q → 1−in the above Theorem we get the following result proved by Annamalai[3]

Corollary 2.4. If f ∈ A satisfies $\frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z}$. Then

$\mu \leq \frac{3\beta-1}{4\beta}$

□ 2(3 − 4μ) if, β $\frac{3\beta-1}{4\beta} \leq \mu \leq \frac{3\beta+1}{4\beta}$

2 □□β if, (27)

|a3 −μa2| ≤

□□ −(β2(3 − 4μ)) if $\mu \geq \frac{3\beta+1}{4\beta}$.

III. CONCLUSION

In this paper we found Janowski α-q-spirallike with power senes of the form and its properties.

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