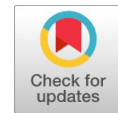


# Cost Sensitivity Ranges in Rough Set Interval Transportation Problem



P. Pandian, K. Kavitha

**Abstract:** A new technique namely, upper-lower bound technique is projected for solving the cost sensitivity analysis of fully rough integer transportation problem. Numerical sample is given to indicate the productivity of the projected technique.

**Keywords:** Transportation problem (TP), rough integer intervals, upper-lower bound algorithm, sensitivity analysis (SA).

## I. INTRODUCTION

The transportation model exposes minimum-cost scheduling problems for transport a goods from journey’s origin to end, such as from business unit to storehouses, or from storehouses to mall, with the transport costs from one place to another being a linear function of number of units transported. The theory of rough sets is obtainable by Pawlak [10]. The rough programming is deliberated by many authors [1, 4, 6, 7 and 12]. Xu and Yao discussed with rough payoffs in which number of players is two. [14]. Youness presented a formulation of rough programming problem which is a non-linear programming with rough set of restrictions [16]. Xu et al proved the proficiency of the rough DEA model [15]. Kundu et al. considered solid transportation model with crisp and rough variables [5]. Subhakanta Dash and Mohanty considered the unit cost of transportation in terms of rough integer interval from journey’s origin to end, [13]. SA in TP is used to acquire facts about how results are affected as the intake data are wide-ranging. SA is most exciting and preoccupying zones in maximization/minimization problems. SA of the optimum result can offer additional information for organization. SA is to analyse the result of the variations of the cost coefficients and the result of variation of the right hand side restrictions on the optimum value of the cost function and the valid upper and lower limits of the results. Cost-sensitive rough set approach is proposed by Hengrong Ju et al. [2]. Shujiao Liao et al. presented cost sensitive attribute reduction problem in DTRS models [11]. In this paper, the idea of solving the Type I cost SA of fully rough integer TP with the help of Upper-Lower bound algorithm. Such algorithm is used to views of sufficiency as the higher level

decision-makers choices. The paper is described as follows: Section 2 and 3 projects the fundamentals of Rough integer interval transportation problem. In section 4, we determine the computation of modi indices and we describe our proposed Upper-Lower bound algorithm. A numerical example is illustrated in Section 5. Final conclusion of the paper in section 6.

## II. PRELIMINARIES

We need the following definitions, which exposed in Hongwei Lu et al. [3].

Let  $D$  indicate the set of all rough intervals on  $R$ . That is,  $D = \{ [b,c], [a,d], a \leq b \leq c \leq d \text{ and } a, b, c \text{ and } d \text{ are in } R \}$

Note that (i) if  $a = b$  and  $c = d$  in  $D$ , then  $D$  becomes the set of all real intervals and (ii) if  $a = b = c = d$  in  $D$ , then  $D$  becomes the set of all real numbers.

### A. Definition:

Let  $A = [[a_2, a_3], [a_1, a_4]]$  and  $B = [[b_2, b_3], [b_1, b_4]]$  be in  $D$ . Then,

$$(i) A \oplus B = [[a_2 + b_2, a_3 + b_3], [a_1 + b_1, a_4 + b_4]];$$

$$(ii) kA = [[ka_2, ka_3], [ka_1, ka_4]] \text{ if } k \text{ is a +ve real interval and}$$

$$(iii) A \otimes B = [[a_2, a_3][b_2, b_3], [a_1, a_4][b_1, b_4]].$$

### B. Definition:

Let  $A = [[a_2, a_3], [a_1, a_4]]$  and  $B = [[b_2, b_3], [b_1, b_4]]$  be in  $D$ . Then,

$$(i) A \leq B \text{ if } a_i \leq b_i, i = 1, 2, 3, 4 ;$$

$$(ii) A \geq B \text{ if } B \leq A, \text{ that is, } a_i \geq b_i, i = 1, 2, 3, 4$$

$$\text{and (iii) } A = B \text{ if } A \leq B \text{ and } B \leq A, \text{ that is, } a_i = b_i, i = 1, 2, 3, 4 .$$

### C. Definition:

Let  $A = [[a_2, a_3], [a_1, a_4]]$  be in  $D$ . Then,  $A$  is said to be non-negative, that is,  $A \geq 0$  if  $a_i \geq 0$ .

### D. Remark:

If  $A = [[a_2, a_3], [a_1, a_4]]$  and  $B = [[b_2, b_3], [b_1, b_4]]$  in  $D$  are non-negative, then,

Manuscript published on 30 August 2019.

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Retrieval Number: J97820881019/19@BEIESP

DOI: 10.35940/ijitee.J9782.0881019

Journal Website: [www.ijitee.org](http://www.ijitee.org)

Published By:

Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

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## Cost Sensitivity Ranges in Rough Set Interval Transportation Problem

$$A \otimes B = [[a_2b_2, a_3b_3], [a_1b_1, a_4b_4]]$$

### E. Definition:

Let  $A = [a_2, a_3], [a_1, a_4]$  be in D, Then, A is called rough integer if  $a_i, i = 1, 2, 3, 4$  are integers.

### III. FULLY ROUGH INTEGER TP

(P1) Minimize

$$[[z_2, z_3], [z_1, z_4]] = \sum_{i=1}^m \sum_{j=1}^n [[c_{ij}^2, c_{ij}^3], [c_{ij}^1, c_{ij}^4]] \otimes [[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]]$$

Subject to

$$\sum_{j=1}^n [[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]] = [[a_i^2, a_i^3], [a_i^1, a_i^4]], i \in I \quad (1)$$

$$\sum_{i=1}^m [[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]] = [[b_j^2, b_j^3], [b_j^1, b_j^4]], j \in J \quad (2)$$

$$x_{ij}^1, x_{ij}^2, x_{ij}^3 \text{ and } x_{ij}^4 \geq 0, i \in I \text{ and } j \in J \text{ are integers} \quad (3)$$

Where

$I = \{1, 2, 3, \dots, m\}, J = \{1, 2, 3, \dots, n\}$ ,  $c_{ij}^1, c_{ij}^2, c_{ij}^3$  and  $c_{ij}^4$  +ve integers are  $\forall i \in I$  and  $j \in J$ ,  $a_i^1, a_i^2, a_i^3$  and  $a_i^4$  are +ve integers  $\forall i \in I$  and  $b_j^1, b_j^2, b_j^3$  and  $b_j^4$  are +ve integers  $\forall j \in J$ . The problem (P1) is called balanced if the entire amount of the supply is equal to the entire amount of the demand.

### A. Definition:

A set of rough intervals  $\{[[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]], \forall i \in I \text{ and } j \in J\}$  is said to be a feasible solution to the problem (P1) if it satisfies the equation (1), (2) and (3).

### B. Definition:

A feasible solution  $\{[[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]], \forall i \in I \text{ and } j \in J\}$  to the problem (P1) is said to be an optimal solution of the problem (P1) if the feasible solution minimizes the cost function of the problem (P1), that is,

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n [[c_{ij}^2, c_{ij}^3], [c_{ij}^1, c_{ij}^4]] \otimes [[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]] \\ & \leq \sum_{i=1}^m \sum_{j=1}^n [[c_{ij}^2, c_{ij}^3], [c_{ij}^1, c_{ij}^4]] \otimes [[u_{ij}^2, u_{ij}^3], [u_{ij}^1, u_{ij}^4]] \end{aligned}$$

$\forall$  feasible  $\{[[u_{ij}^2, u_{ij}^3], [u_{ij}^1, u_{ij}^4]], \text{ for } i \in I \text{ and } j \in J\}$ .

Now, the problem (P1) is divided into (i) uppermost approximation upper bound integer TP (UAUBITP), (ii) lower approximation upper bound integer TP (LAUBITP), (iii) lower approximation lower bound integer TP (LALBITP) (iv) uppermost approximation lower bound integer TP (UALBITP) (UAUBIT)

$$\text{Minimize } z_4 = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^4 x_{ij}^4$$

$$\text{Subject to } \sum_{j=1}^n x_{ij}^4 = a_i^4, i \in I; \quad \sum_{i=1}^m x_{ij}^4 = b_j^4, j \in J;$$

$$x_{ij}^4 \geq 0, i \in I \text{ and } j \in J \text{ and are integers,}$$

(LAUBITP)

$$\text{Minimize } z_3 = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^3 x_{ij}^3$$

$$\text{Subject to } \sum_{j=1}^n x_{ij}^3 = a_i^3, i \in I; \quad \sum_{i=1}^m x_{ij}^3 = b_j^3, j \in J;$$

$$x_{ij}^3 \geq 0, i \in I \text{ and } j \in J \text{ and are integers,}$$

(LALBITP)

$$\text{Minimize } z_2 = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^2 x_{ij}^2$$

$$\text{Subject to } \sum_{j=1}^n x_{ij}^2 = a_i^2, i \in I; \quad \sum_{i=1}^m x_{ij}^2 = b_j^2, j \in J;$$

$$x_{ij}^2 \geq 0, i \in I \text{ and } j \in J \text{ and are integers,}$$

and

(UALBITP)

$$\text{Minimize } z_1 = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^1 x_{ij}^1$$

$$\text{Subject to } \sum_{j=1}^n x_{ij}^1 = a_i^1, i \in I; \quad \sum_{i=1}^m x_{ij}^1 = b_j^1, j \in J;$$

$$x_{ij}^1 \geq 0, i \in I \text{ and } j \in J \text{ and are integers,}$$

Now, we need the following theorem which can be found in [8].

### C. Theorem:

If the set  $\{\bar{x}_{ij}^4, \text{ for all } i \in I \text{ and } j \in J\}$  is an optimum solution for the (UAUBITP) problem of the problem(P1) with the minimal transportation cost  $\bar{z}_4$ , the set  $\{\bar{x}_{ij}^3, \text{ for all } i \in I \text{ and } j \in J\}$  is an optimum solution for the (LAUBITP) problem of the problem(P1) with the minimal transportation cost  $\bar{z}_3$ , the set  $\{\bar{x}_{ij}^2, \text{ for all } i \in I \text{ and } j \in J\}$  is an optimal solution for the (LALBITP) problem of the problem(P1) with the minimal transportation cost  $\bar{z}_2$  and the set  $\{\bar{x}_{ij}^1, \text{ for all } i \in I \text{ and } j \in J\}$  is an optimum solution for the (UALBITP) problem of the problem(P1) with the minimum transportation cost  $\bar{z}_1$ , then the set of rough integer intervals  $\{[[x_{ij}^2, x_{ij}^3], [x_{ij}^1, x_{ij}^4]], \text{ for all } i \in I \text{ and } j \in J\}$  is an optimum solution for the problem (P1) with the minimal transportation cost  $[[\bar{z}_2, \bar{z}_3], [\bar{z}_1, \bar{z}_4]]$  provided

$$\bar{x}_{ij}^1 \leq \bar{x}_{ij}^2 \leq \bar{x}_{ij}^3 \leq \bar{x}_{ij}^4, \text{ for all } i \in I \text{ and } j \in J.$$

#### IV. COMPUTATION OF MODE INDICES

Now, we consider  $c_{ij} + \Delta c_{ij}$  is the perturbed cost coefficient of (i,j)<sup>th</sup> cell, in which  $\Delta c_{ij}$  represents the parameter.

Let  $k = (\text{number of rows } (m) + \text{number of columns } (n)) - \text{number of nonzero basic cells}$

in the Maximal/minimal solution. If  $k = 2$ , we focus the Type I SA then the given rough integer TP is non-degenerate. Therefore, we assume one of the MODI-indices value zero and using the condition  $c_{ij} - (u_i + v_j) = 0$ , for all zero cells (i, j), we can calculate the left over part of the MODI-indices.

##### A. Theorem:

Let (i,j)<sup>th</sup> cell be a non-zero cell corresponding to an maximal/minimal solution of the RSTP with  $\delta_{ij} = c_{ij} - u_i - v_j (\geq 0)$ . If  $c_{ij} + \Delta_{ij}$  is the perturbed cost of  $c_{ij}$ , then the range of  $\Delta_{ij} = [-\delta_{ij}, \infty)$ .

**Proof:** Now, (i,j)<sup>th</sup> cell is a non-zero cell and the perturbed cost  $c_{ij} + \Delta_{ij}$  is not afflicted the current maximal/minimal solution to the problem,  $c_{ij} + \Delta_{ij} - u_i - v_j \geq 0$ . This indicates that,  $\Delta_{ij} \geq -\delta_{ij}$ . Therefore, the range of  $\Delta_{ij} = [-\delta_{ij}, \infty)$ . Hence the theorem.

##### B. Theorem:

Let (i,j)<sup>th</sup> cell be zero cell corresponding to an maximal/minimal solution of the RSTP with  $\delta_{ij} = c_{ij} - u_i - v_j (= 0)$ . If  $c_{ij} + \Delta_{ij}$  is the perturbed value of  $c_{ij}$  and  $U_i$  is the minimal value of  $\delta_{ij}$  for all non-zero cells in the *i*th source,  $V_j$  is the minimal value of  $\delta_{ij}$  for all non-zero cells in the *j*th target, then the range of  $\Delta_{ij} = (-\infty, M_{ij}]$  where  $M_{ij} = \text{the maximal } \{ U_i, V_j \}$  ;

**Proof:** Now, since  $c_{ij} + \Delta_{ij}$  is the perturbed value of  $c_{ij}$  and the current maximal/minimal solution remains optimal,  $\delta_{ij} = c_{ij} - u_i - v_j \geq 0$ , for all non-zero cells in the *i*th source and the *j*th target are positive.

Now, adding the  $\Delta_{ij}$  to  $u_i$ , then  $v_j$ , we have

$$c_{is} - (u_i + \Delta_{ij}) - v_s \geq 0, (i, s) \text{ is non-zero cells, for all } s. ;$$

$$c_{rj} - u_r - (v_j + \Delta_{ij}) \geq 0, (r, j) \text{ is non-zero cells, for all } r.$$

The above relation gives that  $\Delta_{ij} \leq U_i$  and  $\Delta_{ij} \leq V_j$ .

Now, we add any one of the MODI-indices  $u_i$  and  $v_j$ , we take,  $M_{ij} = \text{maximum } \{ U_i, V_j \}$  for attaining enhanced range. Therefore, the range of  $\Delta_{ij} = (-\infty, M_{ij}]$ . Hence the theorem.

#### C. Upper-Lower Bound Technique:

We, now present a new method, namely Upper-Lower bound method found on the Theorem A and the Theorem B to analyse the Type I sensitivity ranges in SARSTP. The Upper-Lower bound method proceeds as follows.

**Step 1.** Calculate an optimum solution to the specified UAUBITP using the Slice-Sum technique [8].

**Step 2.** Calculate the values of the MODI-parameters  $\{ u_i, v_j, i=1,2,\dots,m; j=1,2,\dots,n \}$  using the perception of the Section 4.

**Step 3.** Calculate the MODI parameters table for the optimum solution found in the Step1. and then, find  $\delta_{ij} = c_{ij} - (u_i + v_j)$  for all non-zero cells.

**Step 4.** Calculate the Type I sensitivity ranges of all non-zero cells using the Theorem 4.1. and then, determine the Type I sensitivity ranges of all zero cells using the Theorem 4.2.

**Step 5.** Repeat the steps from 2 to 4 for LAUBITP, LALBITP and LAUBITP with the upper bound(UB) restrictions  $x_{ij}^o \leq y_{ij}^o$ , for all *i* and *j*.

The upper-lower bound technique is exposed the following numerical examples.

#### V. NUMERICAL EXAMPLE

In a medicinal agency, wares are manufactured in 3 business units and it is dispatch to 3 store houses. The least unit shipping cost range from each supply point to each demand point is given below:

**Table-I: Least unit shipping cost**

	S1	S2	S3	Supply
B1	[7,9]	[12,14]	[10,11]	[13,15]
B2	[4,5]	[3,4]	[5,7]	[11,13]
B3	[5,6]	[2,3]	[10,11]	[14,16]
Demand	[20,22]	[11,13]	[7,9]	[38,44]

and the extreme unit shipment cost range from each supply point to each demand point is given below:

**Table-II: Extreme Unit Shipment Cost**

	S1	S2	S3	Supply
B1	[6,10]	[11,15]	[8,12]	[12,16]
B2	[3,6]	[2,7]	[4,9]	[10,14]
B3	[3,7]	[1,4]	[9,12]	[13,18]

## Cost Sensitivity Ranges in Rough Set Interval Transportation Problem

Demand	[19,24]	[10,14]	[6,10]	[35,48]
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Now, the specified problem can be demonstrated as fully rough integer interval TP as follows.

**Table-III: Fully Rough Integer Interval TP**

	S1	S2	S3	Supply
B1	[7,9] [6,10]	[12,14] [11,15]	[10,11] [8,12]	[13,15] [12,16]
B2	[4,5] [3,6]	[3,4] [2,7]	[5,7] [4,9]	[11,13] [10,14]
B3	[5,6] [3,7]	[2,3] [1,4]	[10,11] [9,12]	[14,16] [13,18]
Demand	[20,22] [19,24]	[11,13] [10,14]	[7,9] [6,10]	[38,44] [35,48]

Now, by using step 1. the optimum solution of the specified problem is

$$\begin{aligned}
 & [[\bar{x}_{11}^2, \bar{x}_{11}^3], [\bar{x}_{11}^1, \bar{x}_{11}^4]] = [[6,6], [6,6]], \\
 & [[\bar{x}_{13}^2, \bar{x}_{13}^3], [\bar{x}_{13}^1, \bar{x}_{13}^4]] = [[7,9], [6,10]], \\
 & [[\bar{x}_{21}^2, \bar{x}_{21}^3], [\bar{x}_{21}^1, \bar{x}_{21}^4]] = [[1,1,3], [10,14]], \\
 & [[\bar{x}_{31}^2, \bar{x}_{31}^3], [\bar{x}_{31}^1, \bar{x}_{31}^4]] = [[3,3], [3,4]] \quad \text{and} \\
 & [[\bar{x}_{32}^2, \bar{x}_{32}^3], [\bar{x}_{32}^1, \bar{x}_{32}^4]] = [[1,1,3], [10,14]]
 \end{aligned}$$

With the minimum shipping cost  $[[193, 275], [133, 348]]$ . Now, we consider the Upper Approximation upper bound TP:

**Table-IV: Upper Approximation Upper Bound (UB) TP**

	D1	D2	D3	Supply
F1	[6,10]	[11,15]	[8,12]	[12,16]
F2	[3,6]	[2,7]	[4,9]	[10,14]
F3	[3,7]	[1,4]	[9,12]	[13,18]
Demand	[19,24]	[10,14]	[6,10]	[35,48]

**Table-V: Optimal Solution Of The Upper Approximation UB TP:**

	D1	D2	D3	Supply
F1	10 6	15 10	12	16
F2	6 14	7	9	14
F3	7 4	4 14	12	18
Demand	24	14	10	48

Now, by using the Step 1. The optimal solution to the UAUBITP is

$\bar{x}_{11}^4 = 6, \bar{x}_{13}^4 = 10, \bar{x}_{21}^4 = 14, \bar{x}_{31}^4 = 4$  and  $\bar{x}_{32}^4 = 14$  with the minimum transportation cost is 348.

**Table-VI: Modi Indices Table of the Upper Approximation UB TP:**

	$v_1 = 10$	$v_2 = 7$	$v_3 = 12$	Supply
$u_1 = 0$	10 6	15 10	12	16
$u_2 = -4$	6 14	7	9	14

$u_3 = -3$	7 4	4 14	12	18
Demand	24	14	10	48

Now, by using the Step 3& Step 4, the Type I sensitivity ranges of  $\Delta_{ijk}$ 's of the specified Upper approximation UB

TP are given below:

**Table-VII: Sensitivity Ranges Of Upper Approximation UB TP**

	D1	D2	D3
F1	$(-\infty, 8]$	$[8, \infty)$	$(-\infty, 3]$
F2	$(-\infty, 4]$	$[4, \infty)$	$[1, \infty)$
F3	$(-\infty, 3]$	$(-\infty, 3]$	$[3, \infty)$

Now, we consider the Lower Approximation UB TP:

**Table-VIII: Lower Approximation UB TP**

	D1	D2	D3
F1	6 6	11	8 9
F2	3 13	2	4
F3	3 3	1 13	9

Now, by using the Step 1. The optimal solution to the LAUBITP with the UB restrictions

$x_{ij}^3 \leq \bar{x}_{ij}^4, i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  and are integers, is  $\bar{x}_{11}^3 = 6, \bar{x}_{13}^3 = 9, \bar{x}_{21}^3 = 13, \bar{x}_{31}^3 = 3$  and  $\bar{x}_{32}^3 = 13$  with the minimum transportation cost is 27.

**Table-IX: Modi Indices of the Lower Approximation UB TP:**

	$v_1 = 6$	$v_2 = 4$	$v_3 = 8$
$u_1 = 0$	10 6	15	12 9
$u_2 = -3$	6 13	7	9
$u_3 = -3$	7 3	4 13	12

Now, by using the Step3 & Step 4, the Type I sensitivity ranges of  $\Delta_{ijk}$ 's of the specified Lower Approximation upper bound TP are given below:

**Table-X: sensitivity ranges of lower approximation UB TP**

	D1	D2	D3
F1	$(-\infty, 7]$	$[7, \infty)$	$(-\infty, 4]$
F2	$(-\infty, 1]$	$[1, \infty)$	$[-1, \infty)$

F3	$(-\infty, 4]$	$(-\infty, 4]$	$[4, \infty)$
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**Table-XI: Optimal solution of the Lower Approximation lower bound (LB) TP:**

	D1	D2	D3
F1	7 6	12	10 7
F2	4 11	3	5
F3	5 3	2 11	10

Now, by using the Step 1. The optimal solution to the LAUBITP with the UB restrictions  $x_{ij}^2 \leq \bar{x}_{ij}^3, i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  and are integers, is  $\bar{x}_{11}^2 = 6, \bar{x}_{13}^2 = 7, \bar{x}_{21}^2 = 11, \bar{x}_{31}^2 = 3$  and  $\bar{x}_{32}^2 = 11$  with the minimum transportation cost 193.

**Table-XII: Modi Indices Table of the Lower Approximation LB TP:**

	$v_1 = 7$	$v_2 = 4$	$v_3 = 10$
$u_1 = 0$	7 6	12	10 7

**Table-XIV: Type I sensitivity ranges for Fully Rough Integer ITP**

	D1	D2	D3
F1	$[(-\infty, 8](-\infty, 8], (-\infty, 7](-\infty, 8])$	$[[8, \infty)[8, \infty), [7, \infty), [8, \infty)]$	$[(-\infty, 3](-\infty, 3], (-\infty, 2](-\infty, 4])$
F2	$[(-\infty, 2](-\infty, 2], (-\infty, 1](-\infty, 4])$	$[[2, \infty)[2, \infty), [1, \infty), [4, \infty)]$	$[[-2, \infty)[0, \infty), [-1, \infty), [1, \infty)]$
F3	$[(-\infty, 3](-\infty, 3], (-\infty, 2](-\infty, 4])$	$[(-\infty, 3](-\infty, 3], (-\infty, 2](-\infty, 4])$	$[[3, \infty)[3, \infty), [2, \infty), [4, \infty)]$

**VI. CONCLUSION**

In this paper, the price of transportation from the factories to destinations is deliberated to be rough costs are allocated. Slice-Sum method [8] is proposed for finding the optimal solution for the fully rough interval integer TP. The upper-lower bound technique provides Type I cost SA for the fully rough interval integer TP. The projected technique can be helped an essential implement for the manufacturers when they are approach numerous kinds of logistic models for practical life circumstances having rough integer intermission factors.

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$u_2 = -3$	4 11	3	5
$u_3 = -2$	5 3	2 11	10

Now, by using the Step 3& Step 4, the Type I sensitivity ranges of  $\Delta_{ijk}$ 's of the specified Lower approximation lower bound TP are given below:

**Table-XIII: Sensitivity ranges of lower approximation LB TP**

	D1	D2	D3
F1	$(-\infty, 8]$	$[8, \infty)$	$(-\infty, 2]$
F2	$(-\infty, 2]$	$[2, \infty)$	$[-2, \infty)$
F3	$(-\infty, 2]$	$(-\infty, 2]$	$[2, \infty)$

Likewise, we can discover the ranges of Upper Approximation LBTP. Now the Type I sensitivity ranges for Fully Rough Integer ITP is given below:

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