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Alpha labeling of splitting graph of a gutman tree

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Abstract— ***The splitting graph S'(G) of graph G results by introducing a new point a' with respect to each point a of G such that N(a) equals to N(a') where N(a) and N(a') are the neighborhood sets of a and a' respectively. The Gutman tree is also known as benzenoid tree and caterpillar tree. In this paper, it is proved that a splitting graph of a Gutman tree satisfies an alpha labeling.***

Keywords: Alpha labeling, splitting graph, Gutman tree.

1. INTRODUCTION

Rosa [1] defined an alpha-labeling as a extension of graceful labeling with the additional property that there exists a number η $(0 \le \eta \le E(G))$ such that, for any edge e Ɛ E(G) with terminal vertices a, b Ɛ V(G), $\min[\chi(a), \chi(b)] \le \eta < \max[\chi(a), \chi(b)]$. η is called the critical number of the function χ.

In Gallian's survey [2] it is denoted as S'(G) and in this paper we follow this same notation. Froncek [3] showed that full hexagonal caterpillars admit alpha-labeling. Also in [4], it is shown that straight simple polynomial caterpillars have alpha-labeling. Barrientos and Minion [5] discussed alpha labelings of snake polynominoes and hexagonal chains. Sethuraman and Ragukumar [6] have shown that every tree is a subtree of graceful tree, graceful graph and alpha-labeled graph. Sampathkumar and Wallikar [7] introduced the splitting graph S(G) of a graph G. Combinatorial and physical properties of benzenoid hydrocarbons can be studied via related caterpillars. The bijection between labeled edges of the caterpillar and those of the hexagons of the benzenoid system. Gutman [8] represented such matching relations among hexagons of a benzenoid system compared with the edges of a caterpillar tree. A caterpillar tree is defined to be a tree graph the derivative of which is a path.

In this paper, any Graph is simple, finite and undirected. In general G (V, E) denotes the graph G with vertex set V(G) and edge set E(G) such that the vertex set contains p vertices and edge set contains q edges.

Definition:-

A graph G is said to be graceful if its vertices are labeled from {0, 1, ..., q} and lines (edges) are labeled by the absolute difference of their respective end vertex labels then the resultant edge labels taken all together form the set {1, 2, ..., q}.

II.MAIN RESULT

Theorem

Statement :

Splitting graph of Gutman tree satisfies an alpha-labeling.

Proof:

A Gutman tree can be formed by connecting centers of a finite number of star graphs by an edge. Let p_h, h = 1 to n denote the path vertices of the caterpillar and v_{hb} denote b^{th} end vertex of the star graph whose centre is in p_h. Let q_h denote edges incident at the h^{th} star graph. The edge set q is obtained by $\sum_{h=1}^{n} q_h + (n-1)$.

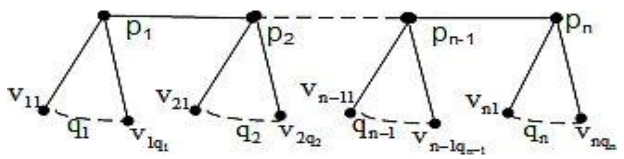

Figure 1: caterpillar graph(Gutman tree)

Splitting graph of caterpillar(Gutman tree) is given below.

The resultant graph S`(G) consists of 2p vertices and 3q edges, which is given below.

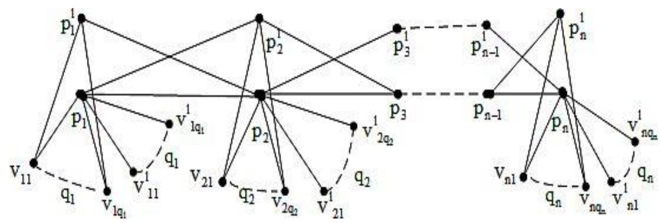

Figure 2: splitting graph of caterpillar(Gutman tree)

Consider the Gutman tree with n stars, in which the star with center p_h has total number of edges $3q_h$ having q_h pendent vertices. The attached pendent vertices are denoted by $v`_{hb}$, b = 1 to q_h and the remaining degree 2 vertices are denoted by v_{hb}, b = 1 to q_h. In the Gutman tree q = $\sum_{h=1}^{n} 3q_h + 3(n-1)$.

vertex labeling are as follows:-

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1st segment:-

$\phi(p_1') = 1$. $\phi(p_1) = 0$. $\phi(v_{1\,b}) = q - q_1 - 2(b-1)$, $b = 1, 2, \ldots, q_1$. $\phi(v_{1\,b}') = q - (b-1)$, $b - q_1$.

2nd segment:-

$\phi(p_2') = q - 3q_1$. $\phi(p_2) = q - 3q_1 - 1$.

$\phi(v_{2\,b}) = 3 + q_2 + 2(b-1)$, $b = 1, 2, \ldots, q_2$.

$\phi(v_{2\,b}') = 2 + (b-1)$, $b = 1, 2, \ldots, q_2$.

Odd segments :-

$\phi(p_{2c+1}') = 3\sum_{h=1}^{c} q_h + 3c + 1$, $c = 1, 2, \ldots, \frac{n-1}{2}$.

$\phi(p_{2c+1}) = 3\sum_{h=1}^{c} q_h + 3c$, $c = 1, 2, \ldots, \frac{n-1}{2}$.

Labeling of odd part of the root branch is as follows:−

$\phi(v_{2c+1\,b}) = q - 3\sum_{h=1}^{c} q_{2h-1} - 3c - q_{2c+1} - 2(b-1)$, $b = 1, 2, \ldots, q_{2c+1}$. $\phi(v_{2c+1\,b}') = q - 3\sum_{h=1}^{c} q_{2h-1} - 3c - q_{2c+1} - (b-1)$, $b = 1, 2, \ldots, q_{2c+1}$, $c = 1, 2, \ldots, \frac{n-1}{2}$.

Even segments :-

$\phi(p_{2c+2}) = q - 3\sum_{h=1}^{c+1} q_{2h-1} - 3c - 1$. $\phi(p_{2c+2}') = q - 3\sum_{h=1}^{c+1} q_{2h-1} - 3c$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

Labeling of even part of the root branch is as follows:−

$\phi(v_{2c+2\,b}') = 3\sum_{h=1}^{c} q_{2h} + 3(c+1) - b$, $b = 1, 2, \ldots, q_{2c+2}$.

$\phi(v_{2c+2\,b}) = 3\sum_{h=1}^{c} q_{2h} + 3(c+1) + q_{2c+2} + 2(b-1)$, $b = 1, 2, \ldots, q_{2c+2}$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

Now, induced edge labeling are as follows:-

1st segment:-

$g(p_1 v_{1\,b}') = q - (b-1)$, $b = 1, 2, \ldots, q_1$. $g(p_1' v_{1\,b}) = q - 1 - q_1 - 2(b-1)$, $b = 1, 2, \ldots, q_1$.

$g(p_1 v_{1\,b}) = q - q_1 - 2(b-1)$, $b = 1, 2, \ldots, q_1$. $g(p_1 p_2') = q - 3q_1$, $g(p_1 p_2) = q - 3q_1 - 1$, $g(p_1' p_2) = q - 3q_1 - 2$.

2nd segment:-

$g(p_2 v_{2\,b}') = q - 3q_1 - 3 - (b-1)$, $b = 1, 2, \ldots, q_2$.

$g(p_2' v_{2\,b}) = q - 3q_1 - (q_2 + 3) - 2(b-1)$, $b = 1, 2, \ldots, q_2$. $g(p_2 v_{2\,b}) = q - 3q_1 - (q_2 + 4) - 2(b-1)$, $b = 1, 2, \ldots, q_2$. $g(p_2' p_3) = q - 3(q_1 + q_2) - 3$, $g(p_2 p_3) = q - 3(q_1 + q_2 - 4$, $g(p_2 p_3') = q - 3(q_1 + q_2) - 5$.

Odd segments :-

Induced labeling of odd part of the root branch is as follows:−

$g(p_{2c+1} v_{2c+1\,b}') = q - 3\sum_{h=1}^{2c} q_h - 6c - (b-1)$, $b = 1, 2, \ldots, q_{2c+1}$, $c = 1, 2, \ldots, \frac{n-1}{2}$. $g(p_{2c+1}' v_{2c+1\,b}) = q - 1 - 3\sum_{h=1}^{2c} q_h - 6c - q_{2c+1} - 2(b-1)$, $b = 1, 2, \ldots, q_{2c+1}$, $c = 1, 2, \ldots, \frac{n-1}{2}$. $g(p_{2c+1} v_{2c+1\,b}) = q - 3\sum_{h=1}^{2c} q_h - 6c - q_{2c+1} - 2(b-1)$, $b = 1, 2, \ldots, q_{2c+1}$, $c = 1, 2, \ldots, \frac{n-1}{2}$.

Induced labeling of even part of the root branch is as follows:−

$g(p_{2c+1} p_{2c+2}') = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+1)$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

$g(p_{2c+1} p_{2c+2}) = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+1) - 1$, $c = 1, 2, \ldots, \frac{n-2}{2}$

$g(p_{2c+1}' p_{2c+2}) = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+1) - 2$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

Even segments :-

Induced labeling of even part of the root branch is as follows:−

$g(p_{2c+2} v_{2c+2\,b}') = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+2) - (b-1)$, $b = 1, 2, \ldots, q_{2c+2}$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

$g(p_{2c+2}' v_{2c+2\,b}) = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+2) - q_{2c+2} - 2(b-1)$, $b = 1, 2, \ldots, q_{2c+2}$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

$g(p_{2c+2} v_{2c+2\,b}) = q - 1 - 3\sum_{h=1}^{2c+1} q_h - 3(c+2) - q_{2c+2} - 2(b-1)$, $b = 1, 2, \ldots, q_{2c+2}$, $c = 1, 2, \ldots, \frac{n-2}{2}$.

Induced labeling of odd part of the root branch is as follows:−

$g(p_{2c+2}' p_{2c+3}) = q - 3\sum_{h=1}^{2c+2} q_h - 3(2c+1)$, $c = 1, 2, \ldots, \frac{n-3}{2}$. $g(p_{2c+2} p_{2c+3}) = q - 3\sum_{h=1}^{2c+2} q_h - 3(2c+1) - 1$, $c = 1, 2, \ldots, \frac{n-3}{2}$. $g(p_{2c+2} p_{2c+3}') = q - 3\sum_{h=1}^{2c+2} q_h - 3(2c+1) - 2$, $c = 1, 2, \ldots, \frac{n-3}{2}$.

In order to prove no two vertices receives same label or no two edges receives same label, the following procedure is given.

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To ϕ is a bijection, consider any two vertices $e_1 = v_{hl}$ and $e_2 = v_{bm}$.

case 1:

let h and b be even, say h = 2c + 2, b = 2c` + 2, l = 2c + 2b and m = 2c` + 2b`. let c` > c. Then $\phi(v_{hl}) - \phi(v_{bm}) = q - 3\sum_{h=1}^{2c+1} q_h - 3(c+2) - q_{2c+1} - l_{b-1} - [\, q - 3\sum_{h=1}^{2c+1} q_h - 3(c`+2) - q_{2c+1} - l_{b`} - 1\,] = 3\sum_{h=2c+2}^{2c`+1} q_h - 3(c` - c) + l_{b`} - b > 0$.

This results $e_1 \neq e_2$ if and only if $\phi(e_1) \neq \phi(e_2)$. Similarly the other cases can be proved. Thus ϕ is a bijection and hence a graceful labeling. To obtain alpha-labeling, the two vertex labels that yield the edge value 1 results in $g(p_n v_{nqn})$ for n is even, $g(p`_n v_{nqn})$ for n is odd.

Example:

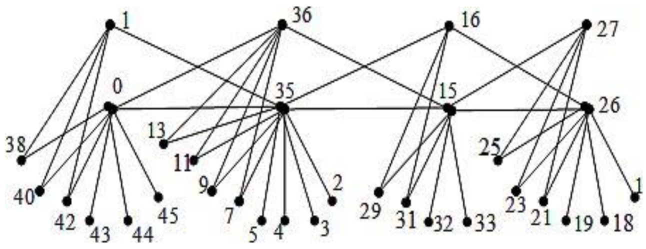

Figure 3: splitting graph of caterpillar with p_4 and lambda = 25

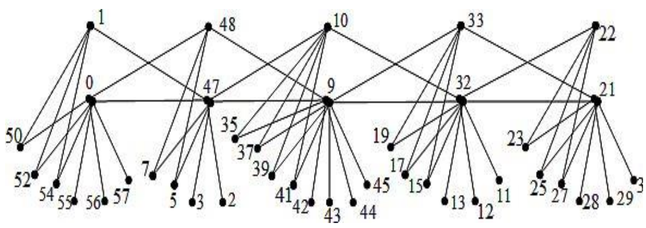

Figure 4: splitting graph of caterpillar with p_5 and lambda = 22

III.CONCLUSION

The Gutman trees are of great importance for understanding and simplifying combinatorial properties of much more complicated graphs in chemical graph theory. These trees resulted from studying the topological properties of benzenoid hydrocarbons. Split of a Gutman tree provides more understaning of the hexagons of the benzenoid system. Hence, we conclude that splitting graph of Gutman tree satisfies α-labeling.

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