

Indoor Positioning System by Using Triangulation Algorithm

Ebru Alp, Tamer Dag, Taner Arsan

Abstract- In this paper, a Wi-Fi based indoor positioning system (IPS) is developed. IPSs are expected to be used in a vast variety of environments such as shopping malls, hospitals, airports and campuses for navigation purposes, real-time location based advertisements, efficient emergency handling situations. Due to the rapid growth of wireless access points in urban areas and the booming usage of smart phones, Wi-Fi has become one of the key technologies to enable location based services for indoors where GPS technology would not work. This paper introduces least square method based triangulation algorithm for IPSs. The implemented system has been tested under various circumstances in order to achieve the minimum error possible. Wi-Fi channel optimization, filtering, calibration of the relation between the signal strength and distance, using more Wi-Fi modems and the least square method are some of the improvements made on the implemented system. The results show that the location accuracy is significantly improved when compared with the simple triangulation algorithm.

Index Terms—Indoor Positioning Development, Triangulation Algorithm, Least Square Method

I. INTRODUCTION

Smartphones are now at the center of our daily life. Thanks to rapid development in smartphone technology, now it is not a sole source of communication device but it is more like a personalized entertainment device, a navigator, mobile internet device and much more in a compact device. Position detection is one of the key areas that many researches have been focused on due to its great importance for navigation and other commercial newly use like real-time location based mobile advertisements. Position detection in outdoor applications can be much easier thanks to GPS technology and, therefore, it has been almost a prevalent practice in the industry. But GPS signal is not available in indoor area due to signal attenuation caused by construction materials and that is why it cannot be used in indoor applications. There are several positioning systems developed for indoor applications but still there is no standard such as the case in outdoor positioning. On the other hand, indoor positioning system of triangulation works with the principle of the GPS system. Therefore, this algorithm also uses the same algorithm with the GPS triangulation. The only difference from the Wi-Fi technology, GPS technology uses satellites but Wi-Fi technology uses access points.

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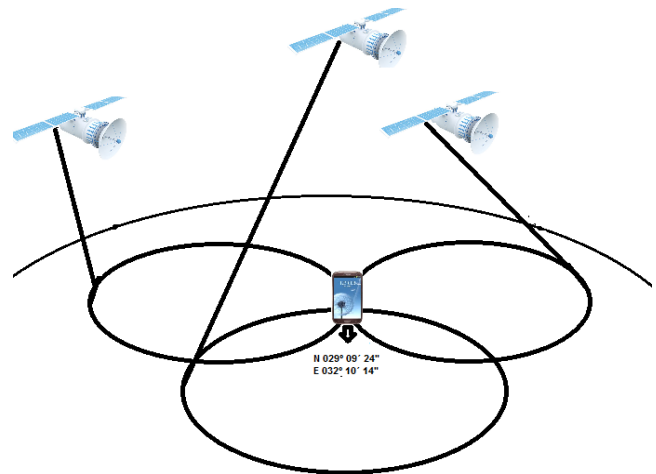


Fig. 1. An example of GPS positioning using 3 satellites

As shown in Fig. 1, 3 satellites examined when the determining the position of the mobile phone. This phone is the located in the area of the intersection of the signal emitted from the satellite. When the location of the phone is determined, triangulation algorithm is used in outdoor positioning system. Also GSM companies calculate the location information using triangulation algorithm with the base station is connected to mobile and surrounding base stations. On the related of the software must be installed on the mobile device in this system[1]. For the unknown position detection, the position of the base stations and the communication with the mobile device is so important. At least three base stations in the GSM location detection technology are considered the reference point (tower point). The position of the unknown object is approximately determined through a mathematical algorithm using the coordinate values of these points. The only difference is the GSM algorithm (cellular communication network) is long distance wireless technology. However, Wi-Fi is middle distance wireless technology. Hence, these two technology works with geometric triangulation algorithm [2].

II. TRIANGULATION ALGORITHM FOR IPSs

1.1 Triangulation Algorithm

Triangulation algorithm is basically based on determining an object approximate location through at least three known locations. The Fig. 2 shows an example of triangulation method where access points A, B and C locations are known (x_A, y_A) , (x_B, y_B) , (x_C, y_C) respectively and the location of the smartphone is unknown (x_0, y_0) . Each access point emits a signal which creates a circle around them. The coordination of smartphone can be obtained by Triangulation algorithm shown on Fig. 2.



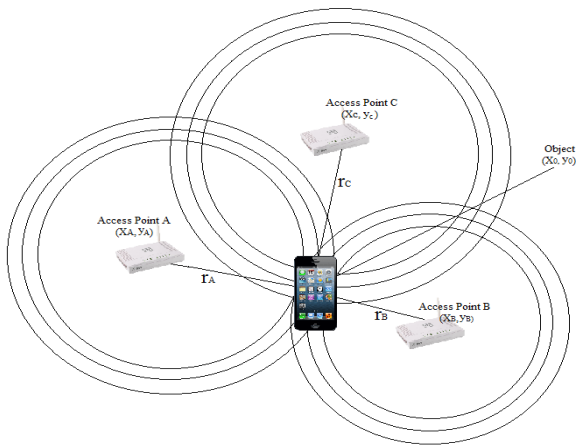


Fig. 2. Three fixed access points and smartphone with unknown location

As shown in figure xxx, access point A's coordinates are x_A, y_A ; access point B's coordinates are x_B, y_B and access point C's coordinates are x_C, y_C . r_A, r_B and r_C is the distance from the access point of the object. The location of the object is unknown and the coordinates of object in figure xxx are shown as x_0 and y_0 . First of all, all distance is calculated from the access points:

$$\begin{aligned} r_A &= \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2} \\ r_B &= \sqrt{(x_B - x_0)^2 + (y_B - y_0)^2} \\ r_C &= \sqrt{(x_C - x_0)^2 + (y_C - y_0)^2} \end{aligned}$$

then,

$$(r_A)^2 - (r_B)^2 = -2x_Ax_0 - 2y_Ay_0 - x_B^2 - y_B^2 + 2x_Bx_0 + 2y_By_0 + x_A^2 + y_A^2 \quad (2.1)$$

$$(r_A)^2 - (r_C)^2 = -2x_Ax_0 - 2y_Ay_0 - x_C^2 - y_C^2 + 2x_Cx_0 + 2y_Cy_0 + x_A^2 + y_A^2$$

From the (2.1) to obtain

$$\begin{bmatrix} (r_A)^2 - (r_B)^2 + (x_B^2 + y_B^2 - x_A^2 - y_A^2) \\ (r_A)^2 - (r_C)^2 + (x_C^2 + y_C^2 - x_A^2 - y_A^2) \\ 2(x_B - x_A) \quad 2(y_B - y_A) \\ 2(x_C - x_A) \quad 2(y_C - y_A) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (2.2)$$

Equations (2.2) can be used to extend for N pieces of sensor target as

$$\begin{bmatrix} (r_1)^2 - (r_2)^2 + (x_2^2 + y_2^2 - x_1^2 - y_1^2) \\ (r_1)^2 - (r_3)^2 + (x_3^2 + y_3^2 - x_1^2 - y_1^2) \\ \vdots \\ (r_1)^2 - (r_N)^2 + (x_N^2 + y_N^2 - x_1^2 - y_1^2) \\ 2(x_2 - x_1) \quad 2(y_2 - y_1) \\ 2(x_3 - x_1) \quad 2(y_3 - y_1) \\ \vdots \\ 2(x_N - x_1) \quad 2(y_N - y_1) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (2.3)$$

define

$$\bar{A} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \quad (2.4)$$

$$\bar{B} = \begin{bmatrix} (r_1)^2 - (r_2)^2 + (x_2^2 + y_2^2 - x_1^2 - y_1^2) \\ (r_1)^2 - (r_3)^2 + (x_3^2 + y_3^2 - x_1^2 - y_1^2) \\ \vdots \\ (r_1)^2 - (r_N)^2 + (x_N^2 + y_N^2 - x_1^2 - y_1^2) \end{bmatrix}$$

Hence the object's coordinates are concluded as

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = (\bar{A}^T \bar{A})^{-1} * (\bar{A}^T \bar{B}) \quad (2.5)$$

Finally, solve (2.5), the object's coordinate (x_0, y_0) can be obtained[3]. Several examinations carried out to decide on the test environment at Kadir Has University Cibali Center Campus to minimize the effects that might weaken Wi-Fi signals and finally a 36m² square area (6m x 6m) was selected for the test environment due to its construction material. AirTies 5341 wireless modems were used in the test. The signal strength of all modems was set to 100%. Three wireless modems were placed on the corners of square room with locations are (0m,0m), (0m,6m), (6m,0m) respectively. The test area was then divided into 40cm² square small areas (15x15 squares) and squares on each corner are not included and kept for just wireless modems. Therefore, 27,04m² (5,2m x 5,2m) test area is obtained. Total 196 points were identified in that 27,04m² test area with 40cm intervals. Signal strength measurement was made by smartphone. To do that Media Access Control (MAC) addresses of all three modems were registered in a Samsung Galaxy S3 branded smartphone and hence smartphone and wireless modem connection was set up. Signal strength of each wireless modem was measured for a period of 12 seconds on each 196 points and was recorded in a text file. Received Signal Strength Indicator (RSSI) is defined as the value measured at the receiving device from signal emitted by the transmitter and measured in dBm. RSSI can be fluctuated very quickly by time since it can be easily affected by the environment. RSSI values also depends on the receiver gain, sensitivity and the signal power measured in the channel and that is the main reason why it cannot provide exact location information. However the location information provided by RSSI value is still adequate for some applications. The location of any unknown object can be estimated using RSSI value if RSSI can be converted into distance. This can be done using log normal formula. Log formula is widely used for indoor positioning studies to calculate RSSI value[4]. In this study the log formula was used to calculate the distance instead.

$$RSSI \text{ (dBm)} = -(10n \log_{10}(d) + A) \quad (2.6)$$

$$d = 10^{\left(\frac{RSSI - A}{10 * n}\right)} \quad (2.7)$$

RSSI: is the value received by the smartphone (dBm)

d: distance in meters

A: received signal strength in dBm at 1 meter

n: propagation constant or path-loss exponent[5]

In calculation of distance separate calculation is made for each wireless modem. The value of "n" is fixed at 2,18 as it is an indoor application. However the value of "A" is not constant for each modem and measured at 1 meter away from each modem respectively. The same calculations are made for each 196 points and each distance value is obtained for algorithm.

1.2 Triangulation Results with Real Data

The test procedure can be summarized as follows:

1. Access points were placed in corner of the test area. ((0,0), (0,600), (600,0)cm)
2. RSSI was measured for 196 points with smart phone



3. The average RSSI was calculated for each point and for each access point.
4. Each access point's RSSI value were measured at 1 meter for value of A
5. Distance value was obtained to RSSI value with log normal
6. This measurement is run triangulation algorithm.

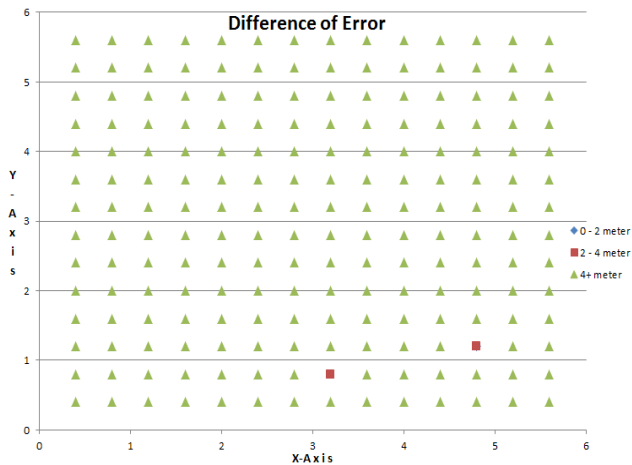


Fig. 3. Location Accuracy under Real Data

Errors are classified at all points at the Fig. 3. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. The class of blue which is 0 – 2 meter error is not in the error distribution figure because outputs have not included any 0 – 2 meter error. When all values are analyzed, table 1 is obtained.

Table 1. The result with real data

Average Error	8,739 m
Minimum Error	3,295 m
Maximum Error	13,515 m

When all results are analyzed it is showed that the estimated point was determined too far from the real point that it should be. The distance between the access points and the receiver have a direct effect on the performance as transmission speed decreases by increased distance. This could be the main reason for such a result, but however this is not clearly observed in the measurements since the value of Transmitter power, two-antenna location to each other, received antenna power and receiver sensitivity is among the important parameters for getting a good performance in transmitting signals for wireless networks. However, at the most important is the structure of building. This is particularly important for indoor applications. Brick walls and metal parts significantly intersect the signal unlike glass and wood which are more permeable[6]. Another important parameter is electronic interference induced by other electronic devices in the same area such as micro-wave ovens. Due to the many disturbances affecting the performance of indoor positioning it is useful to apply some improvements so that the accuracy can be further increased.

1.3 Triangulation Results with Synthetic Data

RSSI value decreases with distance increase from the access point but it is not possible in measurement area because of noise and signal overlapping. Therefore, synthetic data are

generated depending on the distance and this operation is done with log normal formula.

$$RSSI \text{ (dBm)} = - 10n \log(d) + A \quad (2.8)$$

The value of n is taken as a constant 2.18 in the above log normal formula and the values of A are as follows for each access point:

- Value of A for Acp 1: -52
- Value of A for Acp 2: -50
- Value of A for Acp 3: -54
- Value of A for Acp 4: -50

Thus, synthetic RSSI value was obtained for each access point. To examine the results of synthetic data, distance values were obtained of the RSSI values with log normal formula and the obtained data were run through triangulation algorithm.

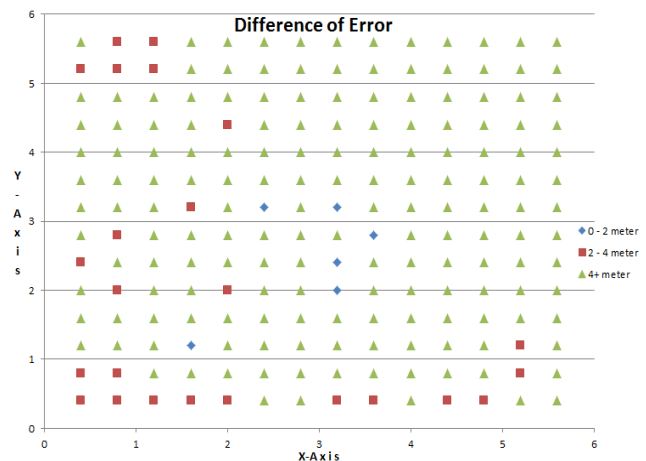


Fig. 4. Location Accuracy under Error-free Synthetic Data

As seen of Fig.4, results of the test environment data indicate more accurate position. Errors are classified at all points at this figure. These classifications are: 0 - 2 meter and 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. Compared with results which they are obtained in front of Kadir HAS University Rezan HAS museum (real data), this output average error 21% were more accurate results. When the analyzed output with synthetic data, table 2 is obtained:

Table 2. The result with error free synthetic data

Average Error	6,931 m
Minimum Error	1,201 m
Maximum Error	10,827 m

In probability theory, the normal distribution is a very common continuous probability distribution. It is synonyms for Gaussian distribution. This distribution is important in statistics. It is often used in the natural and social sciences to represent real-valued random variables whose distributions are not known [7]. A normal distribution has a bell-shaped density curve described by its mean μ and standard deviation σ .



The density curve is symmetrical, centered about its mean, with its spread determined by its standard deviation [8]. Normal distribution calculates by following formula:

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (2.9)$$

However NORMINV standard deviations excel formula is used for easily generating dataset in measurement area.

Formula is used as follows:

NORMINV (probability, mean, standard deviation)

Probability: A probability corresponding to the normal distribution.

Mean: The arithmetic mean of the distribution.

Standard deviation: The standard deviation of the distribution [9] To examine the signal overlap in 5 dbm, the synthetic data calculated by the following formula in excel document:

$$=NORMINV (RAND (); 0; 5) \quad (2.10)$$

In this measurement, it was prepared as a new dataset the following formula:

$$RSSI (dBm) = (-10n \log(d) + A) + NORMINV (RAND (); 0; 5) \quad (2.11)$$

Therefore, the only difference from the synthetic data set is NORMINV (RAND (); 0; 5) calculation. When the triangulation algorithm runs with obtained data, fig.5 is obtained.

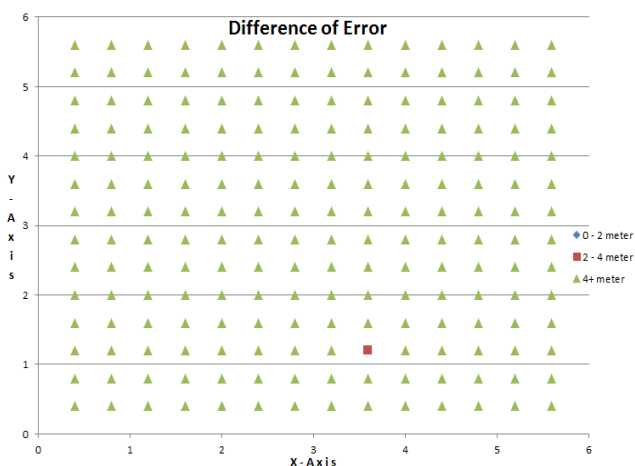


Fig. 5. Location Accuracy under Synthetic Data with 5 dBm

As seen of Fig.5 errors are classified at all points. These classifications are: 0 - 2 meter, 2 - 4 meter and 4+ meter. Each class is represented by a different shape and color. Like triangulation with real data output, the class of blue which is 0 - 2 meter error is not in the error distribution figure because outputs have not included any 0 - 2 meter error. Although A and n are the same in error-free synthetic dataset in this measurement, the result of the 5 dBm test environment data results was too much affected negatively. It showed that, signal overlap in 5dBm is more inaccurate than error free synthetic data. When the analyzed synthetic data with 5 dBm output, table 3 is obtained:

Table 3. The result with error free synthetic data

Average Error	9,797 m
Minimum Error	3,488 m
Maximum Error	15,180 m

III. TRIANGULATION ALGORITHM WITH LEAST SQUARE METHOD

1.4 Least Square Method

The dataset obtained from experimental studies are point. Without a continuous function of these points, the only point does not anything. It is not possible to have the definition of a continuous function which could direct the desired value dataset. However, it is necessary to uncover the functions finds the closest value. Best fitting function is called regression analysis to find the process to a data table. While regression analysis using, the most widely used method is the least squares method. In this method, depending on the actual equation with each other's closest connection between the two values changing. Actual regression model and data generation process should be ideal for this method to be more accurate because most of the data set does not provide the ideal conditions. However, it can create a benchmark point. Therefore knowing ideal conditions is very important for using this method. So status can be determined whether requirements are met and the formation of the deviation can be minimized [10]. Least Square method is to produce a continuous function of the existing dataset. With this method, erroneous datasets obtained for unknown points minimized. Consider first the following model to describe the least square method:

$$\varepsilon = Y - (\beta_0 + \beta_1 X) \quad (3.1)$$

Here;

Y: Dependent variable

X: Independent variable

ε : Random term

β_0 : Invariable (the point where the regression line cut the y-axis)

β_1 : Regression coefficient. (The slope of the regression line)

ε , the obtained values of Y,

$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ represents the vertical distance on the truth. If β_0 and β_1 is known in this equation, it would have to be done to find the $\sum \varepsilon^2$.

3.1 equality in terms of convenience

$$L = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2 \quad (3.2)$$

can define form. n indicates the number of the data obtained. L is representing the sum of the squared in this equation. Conditions that need to minimize L, L's partial derivatives are zero for each parameter (β_0, β_1).

$$\frac{dL}{d\beta_0} = 2\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)(-1) = 0 \quad (3.3)$$

and

$$\frac{dL}{d\beta_1} = 2\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)(X_i) = 0 \quad (3.4)$$

obtained. It would be enough to solve the 3.5 equation for obtain β_0 and β_1

$$\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i) = 0 \quad (3.5)$$

If we expand on both sides of the 4.5 equation with X_i ,

$$\sum_{i=1}^n (X_i Y_i - X_i \beta_0 - \beta_1 X_i^2) = 0 \quad (3.6)$$

From here



$$n\beta_0 = \beta_1 \sum_{i=1}^n X_i = \sum_{i=1}^n Y_i \quad (3.7)$$

and

$$\beta_0 \sum_{i=1}^n X_i + \beta_1 \sum_{i=1}^n X_i^2 = \sum_{i=1}^n X_i Y_i \quad (3.8)$$

obtained.

The solution of 4.7 and 4.8 equations, least square estimator BO and B1,

$$\hat{\beta}_1 = \frac{\sum X_i Y_i - (\sum X_i)(\sum Y_i)/n}{\sum X_i^2 - (\sum X_i)^2/n} \quad (3.9)$$

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \quad (3.10)$$

calculated like this.

From here,

$$S_{xy} = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) \quad (3.11)$$

and

$$S_{xx} = \sum_{i=1}^n (X_i - \bar{X})^2 \quad (3.12)$$

If written form

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad (3.13)$$

as it has.

Linear model should be examined how fit the data with the obtained formula. That's why, epsilon needs to be minimized. For minimizing ϵ (The difference between observations and models that is in formula 3.14), the presence of the best regression model will provide appropriate data. Finding the best regression model in least square method, the sum of the square error occurs to make the smallest in formula 3.15

$$\epsilon_i = Y_i - \hat{Y}_i \quad (3.14)$$

$$\sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \min. \quad (3.15)$$

When triangulation algorithm used to analyze values, it is observed that the data is scattered. The reason is that as the distance from the access point is not in proportion to the signal. Therefore, in this thesis least square method and triangulation algorithm are used together to reduce errors in the triangulation algorithm[11].

1.5 Channel Fixing and Measurement

To avoid the signal conflict, wireless transmitting devices must be operated at least used channel. Herein, referred to above, the 5 MHz share between the two channels can be considered to avoid conflicts that may occur. However, these frequencies are affected by the environment. Therefore 1, 6 and 11 channels are separated theoretically by a wide range that will not interfere to each other. Thus, channel switching and channel retention operation is performed by the number of access points to be used. This channel fixing and channel changing operation are performed according to the number of access points to be used. This process has the basic settings of the wireless router. Therefore increasing the quality of the received signal is provided [12], [13]. Although channel settings can be seen as 13 pieces, it should be at least 3 channels between the 2 different channels. However 3 channel spacing is not very functional because homes are too close or there are too many devices in the shopping malls. This is why some countries have proposed as 5 channels spacing. Therefore, that is more logical use of 1, 5, 9 and 13 channels. In this thesis, a new measurement was made with channel fixing for obtained more accurate signals. This measurement was performed in the same test environment with the previous measurement. In this measurement access points were placed in all corners of the test environment. 4

access points are used which positions are (0, 0), (0,600), (600, 0) and (600,600) cm. Thus, values obtained from the four-access point to the same point. Also in this measurement, 1, 5, 9 and 13 channels which providing at least intersections with the frequency of each other are used to reduce the margin of error. According to the previous measurements, which is fixing channel 6, signals are overlapped to a lesser extent. Therefore, it is planned to be reduced the error percentage.

1.6 Optimized A Values

The value needed for the lognormal formula, each access point provides an average of the RSSI value of 1 meter. However, signal distortions, noise and signal conflicts occur during the measurement in the test environment. Therefore, the error factor of the high rise, the value is measured for 12 seconds not during the measurement period. The training phase is the same as the previous test environment. The only thing that is different to the value used in the measurement process, HTC One brand another smart phone was placed the center of the test environment. The coordinates of this phone are (300, 300) cm. During the measurement process, the software ran on this phone and received RSSI value of all the access points. Application is stopped in the phone at the end of the measurement process. The average of the RSSI values, which is different for each access point, was recorded for each access point and the average RSSI values were obtained. Thus, using most appropriate for environment, optimized A values were calculated with this measurement.

1.7 Brute Force and Optimized n Values

Brute force technique carried out with malicious bots developed by attackers. To avoid this attack, first of all passwords must be include numbers, letters and signs. Secondly, if it is possible completely automated public Turing test to tell computers and humans apart (captcha) security which aim of the project is to identify the behavior of people with computers is protect must be integrate. Thus the user is prompted to enter the security code. This will be prevented from brute force attacks. So, random security code is not estimated by program or malicious bots and it is prevented from brute force attacks.

- This attack works fast in the local system.
- It maintains in multiple attack methods.
- It can be applied to many systems.
- It tries all combinations with the available data [14], [15].

In this thesis, brute force technique was used for optimum n values. Because of the calculating the optimum value, the following steps must be apply for each point and these processes are very difficult to do without the use of an attack program.

For optimum n values with brute force algorithm:

1. Distance is calculated from the relevant access point for each point.
2. Starting from n=0, the estimated distance is calculated equals to real distance for each RSSI values in the text files.

3. Estimated distance and real distance value calculation is stopped when real and estimated distances equal each other.
4. The average of all values is calculated and n value is obtained for the corresponding point.
5. The same process is done to the same access point for each point and the optimum n is obtained with calculating average all of n values.

Previous measurements were used at a fixed value of n. As a value, the value of n is calculated according to the environment, this development is expected to reduce the margin of error. Thus, the environment variable of n was calculated using brute force optimization.

1.8 Least Square Method with Triangulation Algorithm

Data that are obtained in the training process are points. Since these points are not connected with each other, they are not meaningful alone. It may not be a continuous function of these points will be used but a continuous function must be constituted. Considering the signals received from the access point, it showed a decrease in proportion to the farther signal from the access point. The reason is that signal is affected by the environment. In this case, signal strength is less than expected, which makes it difficult to identify the location. Therefore, least square method was used in this thesis. To compare the single triangulation algorithm result, algorithm is run with access point 1, 2 and 3 dataset. Therefore with least square method, a reduction calculated in proportion to the distance from the access point. Thus, it aimed to decrease the error percentage.

1.9 Triangulation Algorithm with Least Square Method Results – 3 Access Points

1.9.1 The result with real data

For compare the single triangulation algorithm result, this algorithm is run for access point (0, 0), (0,600), (600, 0) cm. It seems that output was varying when comparing simple triangulation output.

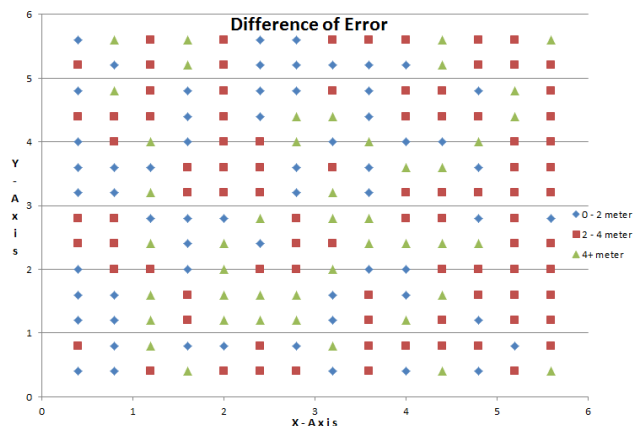


Fig. 6. Location Accuracy under Real Data

Fig. 6. illustrates the location accuracy for every measurement point. Three distinct classifications are made for this purpose. A location accuracy of 0-2 meters can be identified by a green triangle, 2-4 meters by a red square and 4+ meters by a blue diamond. The triangulation algorithm without any enhancements is not very promising. In this environment, the average location error turns out to be 2.826 m as shown in Table 4.

Table 4. The result with real data

Average Error	2,826 m
Minimum Error	0,004 m
Maximum Error	7,513 m

The difference from the previous measurement:

- Channel fixing
- Optimized A and n values
- Least square method: In the first measurement results were calculated in meters but in the second measurement, least square method was used and results were calculated units. In this method 1 unit were calculated as $40\sqrt{2}$ cm. Taken from the program output, it was multiplied by $40\sqrt{2}$. 56,56 were used as an approximate value of $40\sqrt{2}$ and so the location was found to be more accurate. In addition to all the improvements made when it is described, location was detected within average 2,826 m error. Thus, in $600\text{ cm} * 600\text{ cm} = 360.000\text{ cm}^2 = 36\text{ m}^2$ measurement area, position has been detected minimum 7,513 m and maximum 7,513 m error.

1.9.2 The result with error free synthetic data

When the triangulation algorithm with least square method runs with synthetic data which is used in triangulation algorithm with synthetic data, the success of the algorithm was seen in output.

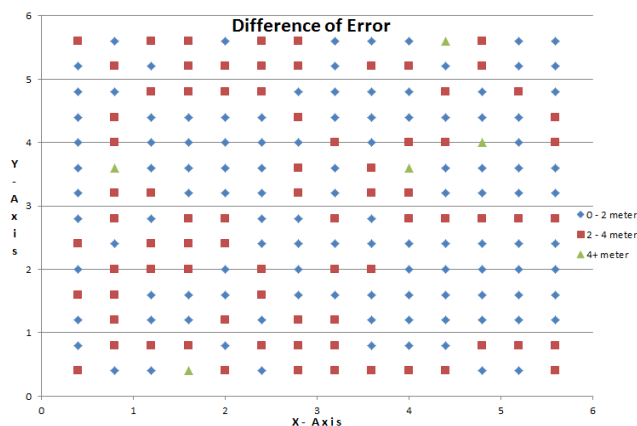


Fig. 7. Location Accuracy under Error-free Synthetic Data

If we have a perfect environment where there is no noise and perfect measurements are made, the triangulation will perform as shown in Fig. 7. When this result is analyzed table 5 is obtained:

Table 5. The result with error free synthetic data

Average Error	1,909 m
Minimum Error	0,165 m
Maximum Error	4,965 m



1.9.3 The result with synthetic data with 5 dBm

When the algorithm runs with synthetic data with 5dBm, output shows that additional improvements in triangulation algorithm are very successful to compare the single triangulation algorithm with synthetic data with 5dBm.

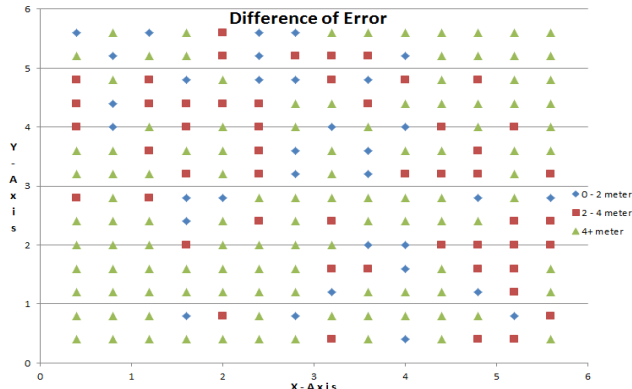


Fig. 8. Location Accuracy under Synthetic Data with 5 dBm

Like shows in Fig.8, when all values are analyzed, the estimated point was determined to be close compare the single triangulation algorithm with synthetic data with 5dBm. Table 6 shows that all average error meter in triangulation algorithm with least square method with synthetic data with 5 dBm.

Table 6.The result with synthetic data with 5 dBm

Average Error	4,881 m
Minimum Error	0,414 m
Maximum Error	11,325 m

IV. CONCLUSION

According to the results of all measurements in this thesis; In measurements made with simple triangulation algorithm output (without any development in triangulation algorithm and measurement feature), it is determined that there are too many errors. The result of the measurements with least square method development in triangulation algorithm and measurement feature, it was seen that error was too low. Errors free dataset outputs showed minimum error according to real measurements outputs, because there has been no signal overlapping in test environment. With least square developed algorithm. Synthetic dataset with 5 dBm output shows that, signal overlapping in noisy area (5 dBm). In these measurements, they have been showed that, signal strength is not decreased in proportion to distance from the signal access point for that signal overlapping. Results are compared with fuzzy algorithm; fuzzy algorithm indoor location is seen that the location is less defective [3]. However, 4 access points must be in measurement area for fuzzy algorithm. These access points should be placed on a square measurement area. There is no such necessity for triangulation like fuzzy. Triangulation algorithm is done regardless of the geometry of the measuring area with a minimum of 3 access points. When simplify the above table to make it more understandable, table 11 is obtained. This table shows that triangulation with least square method is the best indoor positioning system in this thesis.

Table 11.Average error meters of all outputs

	Average error meter		
	Synthetic 0 dBm	Synthetic with 5 dBm	Real
Triangulation	6,931	9,797	8,739
Triangulation + LSM	1,909	4,882	2,826

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