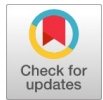


# Interactions of a Deformable Wheel with a Deformable Support Surface

Kovbasa Volodymyr, Priliepo Nataliia



**Abstract:** *The article analyzes methods for solving problems on the drive systems' compacting effect on the support surface (soil). There are analyzed results of studies on the distribution of pressures and tensions in the contact surfaces depending on the propulsors' mechanical properties and the support surface's material. The article also includes an analysis of methods for formalizing contact surfaces and constructing mathematical models for determining the distribution of pressures, deformations, and tensions in the contact surfaces. Numerical methods for solving contact problems that have been used recently are not integral. To obtain such solutions, numerical FEM and DEM methods can be used for a specific problem, i.e. for a problem with a known geometric data source and specific mechanical properties of a contacting body. Based on the studies conducted, it was concluded that to understand the interaction of a deforming wheel with a deformable surface, it is necessary to use an analytical method for solving contact problems in a three-dimensional setting as the most general and productive. It'll allow us to determine the geometrical dimensions and shapes of the contact spot, determine dependencies of the geometrical parameters of the wheel, rolling resistance, tension-strain state of the contacting surfaces depending on the loading conditions of the wheel, its mechanical properties, and the support surface.*

**Keywords:** *Deforming Wheel, Support Surface, Soil Compaction, Pressure and Tension in the Contact Surface, Contact Zones of Deformable Bodies.*

## I. INTRODUCTION

It has been established that in conditions of intensification of agricultural production, multiple passages of technological and transport machines on the field lead to over-compaction of soil, arable and sub-arable horizons, and the destruction of soil's upper layers' structure, which leads to deterioration of its agrophysical and mechanical properties, increasing of bulk mass, decreasing of porosity, aeration, and, as a result, decreasing of soil fertility.

Despite many years of research being conducted on the problem of soil compaction, scientists have no consensus on the choice of criteria for assessing the compressive impact of drive systems. The most complete justification for choosing such criteria can be found in the works of I. Ksenevich, V.

Skotnikov, M. Lyasko, V. Rusanov, O. Kononov, etc. These works indicate that soil density makes a decisive impact on crop harvests. Although there are proposed solutions, the numerical characteristics of the compaction influence of permissible levels still do not fully explain the ecological compatibility of such complex system as «vehicle-technology-soil-nature».

The solution to the problem of soil compaction should be closely connected with the further development of a theory of wheeled and tracked compactor interaction with soil, along with the complex development of rational environmental technologies and machines.

## II. THEORY AND CALCULATIONS

Research in the field of propulsor-soil-plant system, in an initiative or planned manner, was carried out, in different years, by a large group of scientists from Ukraine, Kazakhstan, Lithuania, Moldova, Estonia, Belarus, as well as several other foreign countries.

An attempt to assess the influence of external forces acting on the soil during its processing dates back to the last century. Most fully this problem is highlighted in the work of A. Kononov [1], which is related to the study of the implementation and agro-technical permeability of wheel tractors on loam soil. Even though his work [1] is reduced to a justification of a local brand tractor usage, some of its results are of undisputed interest. For example, when assessing the impact of compactors on the soil, in various combinations, the following parameters were determined: bulk weight, structure, soil temperature and filtration, external forces affecting the soil, and the formation of tracks.

In addition, the effect of various types of compactors on the harvest of several crops was evaluated. The impact of soil density on the harvest was also determined in laboratory conditions. To analyze the interaction of propulsors in the machine's drive systems with soil, the latter must be formalized in the form of a model of a particular environment so that the properties of this model could comply most fully with the properties of the real soil.

According to the type of representation of the soil structure model, all works related to soil deformation can be divided into several groups. The ones where the soil is [2] [16] [17] [18] [19]:

1. solid;
2. continuous elastic medium;
3. continuous incompressible bulk medium (i.e. discrete bulk medium);
4. continuous elastic-viscous medium;
5. continuous elastic-viscoplastic medium.

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At the same time, the next methods are used as an analytical apparatus: method of solid mechanics, method of elasticity theory and its simplified variants, method of soil mechanics, method of probability and dimension theories, method of statistical mechanics, method of rheology [3].

Generally, the supporting surface of pneumatic wheels with appropriate tire treads consists of protrusions and cavities of various configurations [4]. In such cases, the intersection line of the radial plane, which passes through the wheel's axis of rotation and the unloaded external surface of the tire tread under external forces, has a finite radius. These characteristics stated above greatly complicate the methods used to establish the necessary calculated correspondences for determining the distribution of contact pressures of pneumatic tires on the supporting surface. To establish the functions of pressure distribution acting within the contact surface between the wheel and the support surface, it is necessary to have correspondences between the pressure and deformations of the support surface and the pneumatic tire, if its bending, influenced by the existing load is significant. That in particular distinguishes the pneumatic wheel from the rigid gear. We know of two schemes, used to, generally, mathematically describe the interaction between the wheel and the support surface [5]. The first one considers the fixed moment of the wheel rolling along the support surface. In all its variations, it is common that the front and rear parts of the deforming wheel or the front and rear parts of the deforming support surface are different (asymmetric epure of contact pressure distributions, different lengths of tire loading and unloading branches, different level of the support surface in the front and the rear, etc.) [20]. The second scheme deals with the compression of the cylindrical body with an initially flat surface [6].

Both are thoroughly used in dated thematic works with published studies results on determining the distribution of normal, and in specific cases – tangent, pressures, measured on grousers during rolling for different tires on different support surfaces.

Analysis and generalization of these results show that due to the lack of data on the magnitude of real operational margin in all considered works, the results of these works can be evaluated only in qualitative terms.

Significant importance has the connection of pressures that are created in the soil by drive systems with changes in its properties. The most common correlations that associate pressure with soil density are the following [7]:

$$\frac{1}{\rho} = BVW = m \log \sqrt{\sigma_m^2 + \tau_{\max}^2} + n \left( \frac{\tau_{\max}}{\sigma_m} \right) + b, \quad (1)$$

$$\rho = \rho_0 + b \ln \left[ \sigma_m (1 + \tau_{\max}) \right], \quad (2)$$

where  $\rho$  – soil density,  $m, n, b$  – empirical coefficients determined from experiments on soil deformation,  $\sigma_{\max}, \tau_{\max}$  – hydrostatic and maximum tangent pressures.

Relatively recent studies [7]–[9], focusing on the flat contact problem of the interaction between the elastic deformed wheel with an elastic deformed surface, have significantly influenced our research. In these works, the equations of absolute movements of the wheel and surface are given in the form of:

$$u_p[x,0] = -M_p \int_{a_1}^a \left( (-v_{1p} T_\xi) + v_{2p} (-P_\xi + P_m) \right) \frac{Bt}{t^2 + (x+t-\xi)^2} d\xi; \quad (3)$$

$$v_p[x,0] = M_p \int_{a_1}^a \left( (v_{1p} (-P_\xi + P_m) + v_{2p} P_\xi) \right) \frac{Bt}{t^2 + (x+t-\xi)^2} d\xi; \quad (4)$$

$$u_k[x,0] = -M_k \int_{a_1}^a \left( (-v_{1k} T_\xi + v_{2k} (P_\xi - P_m)) \right) \frac{Bt}{t^2 + (x+t-\xi)^2} d\xi; \quad (5)$$

$$v_k[x,0] = M_k \int_{a_1}^a \left( (v_{1k} (P_\xi - P_m) + v_{2k} T_\xi) \right) \frac{Bt}{t^2 + (x+t-\xi)^2} d\xi, \quad (6)$$

where  $u_p[x,0], v_p[x,0], u_k[x,0], v_k[x,0]$  – movement in the direction of the longitudinal and vertical coordinate axis of contacting bodies' surfaces in the contact zone  $\{a_1, a\}$  for the soil and the wheel;  $T_\xi, P_\xi, P_m$  – distributed pressures in the contact zone, tangent and vertical, influenced by the gravity affecting the wheel and the vertical reaction force in the contact zone, created in the zone and caused by the moment of rolling resistance;

$$M_p = \frac{G_p \mu_p}{6G_p(1+\nu_p)} (-1 + e^{\mu_p}), M_k = \frac{G_k \mu_k}{6G_k(1+\nu_k)} (-1 + e^{\mu_k}), \nu_{1p} = 2(-2 + \nu_p), \nu_{2p} = (-1 + 5\nu_p),$$

$\nu_{1k} = 2(-2 + \nu_k), \nu_{2k} = (-1 + 5\nu_k)$  – deformation properties of the wheel and the soil (obtained as a result of all transformations calculated above, after bringing the equations to a two-dimensional form), and directly  $G_p, G_k, \mu_p, \mu_k, \nu_p, \nu_k$  – moduli of elasticity, viscosity coefficients of distortion deformations, and coefficients of lateral expansion (potentially Poisson's ratio) of the soil and the wheel.

With a sufficiently long deformation time, the exponential functions that are included in the deformation constants  $M_p, M_k$ , are converted into unity, and depend only on the moduli of elasticity and the coefficients of lateral expansion;  $t, B = \pi/5b$  – the coefficient that eliminates the singularity of (6) and the coefficient that ensures the fulfillment of conditions on the limit, provided that  $b$  – is the width of the wheel. In the same works [7]–[9], we can find correlations that determine the limits of the wheel-soil contact in the following form:

$$a_1 = - \frac{\sqrt{6} \sqrt{g M_k m \nu_{1k} + \frac{M_k M \nu_{2k}}{r}}}{\sqrt{3r^2 + g M_k m r \nu_{1k} + M_k M \nu_{2k}}}; \quad (7)$$

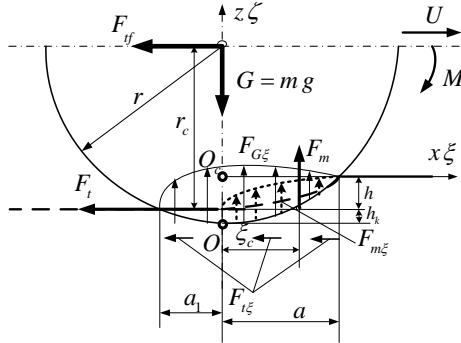
$$a = \frac{\sqrt{2} \sqrt{6M_k M r^2 \nu_{1k} + g M_k m r^3 \nu_{1k} + 6M M_p r^2 \nu_{1p} + g m M_p r^3 \nu_{1p} + M M r^2 \nu_{2k} - M M_p r^2 \nu_{2p}}}{\sqrt{g M_k m r \nu_{1k} + g m M_p r \nu_{1p} + M_k M \nu_{2k} - M M_p \nu_{2p}}}. \quad (8)$$

### III. RESULTS AND DISCUSSION

Recent research has explored numerical approaches to contact problems utilizing Finite and Discrete Element Method techniques [10]–[12].



Unfortunately, these papers do not answer how the geometrical parameters affect the deforming wheel, deformable surface, or the magnitude of the applied force on the contact spot magnitude. Furthermore, these results lack comprehensiveness, limiting their extension to more general cases. For figuring out this problem we want to construct the conditions on the contact surface of a deformable wheel - a deformable support surface under the conditions of inconsistency of the geometric shapes of the contacting surfaces. In the first stage, it is necessary to analyze the distribution of forces in the contact zone between the deformable wheel and the deformable surface. The general scheme of conditions on the contact surface is presented in Fig. 1 [13].



**Fig. 1. General Scheme of the Contact of the Deformable Wheel with the Deformable Surface**

The coordinate system  $xz$  corresponds to the horizontal and vertical surface and wheel coordinate systems, respectively. The wheel with a total radius  $r$  and rolling radius  $r_c$  is loaded by the weight  $G$  brought to the axis of its rotation. This force is distributed over the contact surface by the magnitude of  $F_{G\xi}$ . The point  $O_c$  corresponds to the momentary center of the wheel rotation, the distances  $a_1, a$  determine the rear and front edges of the contact zone between the wheel and the surface.

Under the action of the torque  $M$ , a  $F_t$  driving force is formed and distributed over the contact surface  $F_{t\xi}$ .

The torque creates resistance to the rolling of the wheel, which is distributed over the contact surface  $F_{m\xi}$ . The force  $F_f$  refers to the reduced traction resistance, applied to the wheel axis. Determining the concentrated forces does not cause difficulties, since it is a trivial problem of theoretical mechanics. At the same time, the shapes of the contact surface for the wheel and the deformed support surface will look like this:

$$\zeta_n = h - \frac{\eta^2 k^2}{r} - \frac{l^2 \xi^2}{r}; \zeta_k \rightarrow h_k - \frac{\eta^2 k^2}{r} - \frac{l^2 \xi^2}{r}. \quad (9)$$

In the general case, the method for analytically solving the contact problem, assuming geometric linearity between stresses and strains, involves finding functions that are solutions of elliptic equations.

Such functions are widely known and represented in the form of the Boussinesq - Cerruti solution [14]. For further development, we needed to solve an equation of surface displacement based on applying the biharmonic functions [15].

Based on previously solved equations [15], the equations of movements in the directions  $x \rightarrow u, y \rightarrow v, z \rightarrow w$  will look like this:

$$u = \frac{1}{4\pi G} \left( 2 \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x} + 2\nu \frac{\partial \psi_1}{\partial x} - z \frac{\partial \psi}{\partial x} \right); \quad (10)$$

$$v = \frac{1}{4\pi G} \left( 2 \frac{\partial G}{\partial z} - \frac{\partial H}{\partial y} + 2\nu \frac{\partial \psi_1}{\partial y} - z \frac{\partial \psi}{\partial y} \right); \quad (11)$$

$$w = \frac{1}{4\pi G} \left( 2 \frac{\partial H}{\partial z} + (1 - 2\nu) \psi - z \frac{\partial \psi}{\partial z} \right), \quad (12)$$

where  $G$  – the elasticity modulus of shear strains,  $\nu$  – Poisson’s ratio. Respectively, expressions obtained from (10)–(12) will have a rather lengthy form, as they consider the second and third terms of equations, in this case – irrotational strains and their changes in directions. Analytical integration of these expressions is difficult, therefore, if the irrotational strain components are neglected, then existing calculations can be used [11], [12], [13]. In this case, the movement of the contact surfaces can be described by expressions in the following form:

$$w = \frac{1 - \nu^2}{E} \int_{-b-a}^b \int_{-a}^a \frac{pz[\xi, \eta]}{\sqrt{((x-\xi)^2 + (y-\eta)^2 + (z)^2)}} d\xi d\eta; \quad (13)$$

$$u = \frac{1 - \nu^2}{E} \int_{-b-a}^b \int_{-a}^a \frac{px[\xi, \eta]}{\sqrt{((x-\xi)^2 + (y-\eta)^2 + (z)^2)}} d\xi d\eta; \quad (14)$$

$$v = \frac{1 - \nu^2}{E} \int_{-b-a}^b \int_{-a}^a \frac{py[\xi, \eta]}{\sqrt{((x-\xi)^2 + (y-\eta)^2 + (z)^2)}} d\xi d\eta. \quad (15)$$

However, it’s necessary to point out, that (13)–(15) are relevant provided they are formed under the influence of the forces normal to the contact surface.

#### IV. CONCLUSION

The primary objective of our research is to get a definitive, mathematically based substantiation of the interrelation of the deformable drive wheel with the soil. After analyzing existing studies on the interaction of the wheel propulsor with the soil, we managed to discern the distribution of pressures in the contact zone of the driven deformable wheel with the soil. Now, having solved the contact problem in the general case, we got the components of movements in the  $x, y, z$  directions. The next stage of our research will contain determination of the components of the distribution of forces on the contact surface.

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Authors Contributions	All authors have equal participation in this article.

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